SYRTO

SYstemic Risk TOmography
Signals, Measurements, Transmission Channels, and Policy Interventions

Mutual Funds Dynamics and Economic Predictors

Gianni Amisano, Roberto Savona

SYRTO WORKING PAPER SERIES
Working paper n. 15 | 2014

This project has received funding from the European Union's Seventh Framework Programme for research, technological development and demonstration under grant agreement n° 320270.

This documents reflects only the author's view. The European Union is not liable for any use that may be made of the information contained therein.

Mutual Funds Dynamics and Economic Predictors[†]

Gianni Amisano[*] and **Roberto Savona**[**]
This Draft: September 2014

Abstract

We propose a new Bayesian time-varying CAPM-based beta model to analyse how mutual funds' managers use predictors to change their betas in the short- and medium-run. Our empirical work is based on 5,377 U.S. domestic equity mutual funds over the 1990-2005 period. The main results can be summarized as follows. First, beta dynamics are significantly affected by economic variables, although managers seem not to care about benchmark sensitivities with respect to predictors in choosing their instrument exposure. Second, persistence in beta dynamics and leverage effects are substantial while market timing has a minor role. Third and most important, we find that fund managers tend to converge towards long-run, instrument-based hedging investment strategies and that short-run predictors play a scant role. This offsets the negative market timing and produces positive Jensen's alphas for most mutual fund categories.

Keywords: Equity mutual funds; conditional asset pricing models; time-varying beta; Bayesian analysis
JEL Classification: C11, C13, G12, G13.

[†] This project has received funding from the European Union's Seventh Framework Programme (FP7-SSH/2007-2013) for research, technological development and demonstration under grant agreement no 320270 SYRTO.
[*] Gianni Amisano: DG-Research, European Central Bank. Address: European Central Bank, Kaiserstrasse 29, D-60323, Frankfurt, e-mail: gianni.amisano@ecb.int.
[**] Roberto Savona: Department of Economics and Management, University of Brescia. Address: Dipartimento di Economia e Management, Università degli Studi di Brescia, c/da S. Chiara n° 50, 25122 Brescia (Italy). E-mail: savona@eco.unibs.it. Financial support from the *Newfin Financial Innovation Research Centre* at Bocconi University is gratefully acknowledged.

1. Introduction

In this paper we analyze how mutual funds managers use equity and bond predictors in changing their portfolio structures over time. How do predictors enter into asset allocation decisions and how important are they in the short- and medium-run? How does private and public information enter into market timing activity and fund returns? Our main conjecture is that while economic predictors may explain little in the short-run, mutual fund managers may attribute more importance on the long-run, in this way converging towards instrument-based investment strategies. This is indeed our main finding.

To address the above mentioned questions and test our conjecture, we introduce a novel Bayesian time-varying CAPM-based beta. In this model we assume that managers change their systematic risk exposure in part by observing how benchmark returns are related to a set of predictors, and in part on the basis of their own information set which is unobservable for the econometrician, who basically observes only return patterns over time. When changing their exposure to systematic risk, managers take into account potential benefits arising from market timing, via benchmark predictability, and from private information.

Our approach is inspired by a number of different contributions. First, we draw on Mamaysky *et al*. (2008), who estimate time varying mutual funds alphas and betas with a state space model, but they find that for most funds conditioning upon information does not improve the model's fit, thus implying a scant role for macroeconomic factors in explaining funds dynamics. Our asset pricing model is also similar to that used in Jostova and Philipov (2005), who propose a stochastic mean-reversion beta. As in Pastor and Stambaugh (2009), we develop a framework in which predictors are assumed to be imperfect and innovations in the predictive system are correlated according to priors based on theoretical and empirical features. We therefore follow Ferson and Schadt (1996) and Becker *et al*. (1999) in assessing timing ability within a conditional asset pricing model context. bringing all these elements together, we introduce a Bayesian conditional market timing approach by allowing for correlation among the innovations of portfolio returns, time-varying betas and benchmark returns.

Our empirical analysis is performed on equally weighted portfolios obtained from 5,377 US domestic equity mutual funds over the 1980-2005 period using monthly data. Our main result is that beta dynamics are significantly affected by conditioning variables. Idiosyncratic beta shocks assume a major role in the short-run, thus confirming Mamaysky *et al*. (2008), while in the medium-run predictors play a major role in explaining the total variation in systematic risk dynamics, in sharp contrast with Mamaysky *et al*. (2008). These results suggest the tendency of mutual funds to converge over time towards instrument-based investment strategies. The trend and term spread seem to be the most important predictors as instrument-based rules in beta variations, even though managers do not seem to care about benchmark sensitivities with respect to predictors in choosing their instrument exposure, either in sign or in magnitude.

We also find, by and large, relevant persistence in beta variation, though some significant differences arise due to specific fund styles depending also on predictors. Interestingly, the long-run mean persistence parameters for beta have significant negative correlation, indicating that the lower the long-run mean for beta, the weaker its mean-reversion. This leads us to conclude that dynamic funds with significant beta variation exhibit low long-run beta average.

As for market timing, our Bayesian conditional measure reveals that no mutual fund category shows significantly positive timing ability over the 1990-2005 period: most funds have in fact negative market timing and negative leverage effects which can be interpreted as generated by some sort of hedging. This suggests that fund managers are more focused on long-run hedging strategies rather than on pure aggressive market timing strategies. This interpretation is supported by a structural VAR-based beta variance decomposition analysis that we use to show that mutual funds tend to converge towards instrument-based investment strategies.

The remainder of the paper is organised as follows. Section 2 describes our model. Section 3 describes the estimation procedure and discusses our Bayesian approach. Section 4 presents the data and section 5 reports the empirical results regarding mutual funds' performance and beta dynamics, while Section 6 discusses results on conditional market timing. Section 7 looks at beta dynamics in more depth by exploring the variance decomposition. Section 8 concludes.

2. Fund Dynamics

Following Admati *et al.* (1986), we adopt a parsimonious three-equation representation. For each fund (or portfolio of funds) analysed, the model contains a portfolio return equation, a time-varying rule for benchmark risk exposure and a benchmark return forecasting model. The basic assumption is that managers are single-period investors who maximise the conditional expectation of $u(r_{p,t})$, an increasing, concave objective function that depends on returns in excess of the risk-free rate, $r_{p,t} = R_{p,t} - R_{f,t}$, where $R_{p,t}$ denotes the return of the managed portfolio at time t and $R_{f,t}$ the return of the risk-free asset. The expectation is conditional on a set of *predictive* (or *information* or *conditioning*) variables $\mathbf{z}_{t-1}$, which we assume stationary and we express in demeaned and standardised form, and a private, unobservable signal about the future performance of the market, s_t, which is modelled as a Gaussian variable. As in Becker *et al.* (1999), the maximisation problem is simplified by assuming that portfolio managers choose between the risky market portfolio with return $R_{m,t}$ and the risk-free asset. Hence, we can define $R_{p,t} = R_{f,t} + w_t R_{m,t}$, where w_t is the weight of the market portfolio. In this setting, the portfolio selection problem is

(1) $$\max_{w_t} E\left[u\left(r_{p,t}\right) \middle| \mathbf{z}_t, s_t\right].$$

The solution of the portfolio selection problem is to adjust market exposure w according to $\mathbf{z}$ and s every period t, assuming that the objective function is time-invariant, i.e. that the conditional distribution of the returns $r_{p,t}$, given the predictors and the signal, is time homogenous. Formally, this amounts to:

(2) $$w_t = w\left(\mathbf{z}_t, s_t\right).$$

This implies that the optimal market portfolio weight at t depends on the conditional mean-variance ratio of the tangency portfolio, given the information variables $\mathbf{z}$ and s at t.[‡]

[‡] Becker *et al.* (1999) discuss how to find optimal weights in such a setting. Their approach is based on applying first order conditions on an objective function characterized by constant Rubinstein-type measure of risk aversion (Rubinstein, 1973).

As pointed out by Chen *et al.* (2010), assuming that *w* evolves over time as a linear function of the private signal *s* implies that the return of the managed portfolio with market timing ability should exhibit a convex pattern relative to the returns on the market benchmark. Classical market timing approaches have provided with convincing theoretical reasons for such functional form, which can be expressed as a quadratic function of market returns (Treynor and Mazuy, 1966) or as an "isomorphic correspondence" to some non-linear option strategy pay-off (Merton, 1981). However, this convexity hypothesis is still a controversial point in finance. Indeed, empirical evidence is contradictory and shows anomalous concavity patterns (e.g. Elton, *et al.*, 2012).

Other problems arise from the interaction between selectivity and market timing. Glosten and Jagannathan (1994) pointed out that market timing may be spuriously signalled by negative correlation between the two components if managers take long put option positions which lower the fund's beta when stock returns are low. Grinblatt and Titman (1989) highlighted problems in performance measurement when the fund's beta varies without any active portfolio rebalancing. It is thus clear that modelling beta evolution plays a key role in providing an empirical counterpart to the theoretical model embedded in equations (1) and (2).

Our paper uses a novel approach to achieve this goal, enabling us to explain how expected and unexpected market returns affect expected and unexpected portfolio returns. In our framework, systematic risk exposure is the unobservable fulcrum of a system in which benchmark and fund returns are connected through imperfect economic predictors, i.e. the information variables **z**.

2.1. The Model

Our model generalises the conditional asset pricing approach by assuming that predictors affecting benchmark returns also enter the time varying specification for beta as covariates in a dynamic stochastic equation[§]. This means that the predictors contribute to systematic risk dynamics by affecting the benchmark and in addition they partially determine beta evolution. At one extreme, when predictors are irrelevant, beta evolves in a purely stochastic way. At the other extreme, when the beta shock is absent, i.e. within the classical conditional asset pricing literature which postulates that beta is a deterministic function of a set of conditioning variables, predictors perfectly determine beta. Our specification can flexibly capture how managers use predictability to modify their systematic risk exposures. It also encompasses the above extremes as special cases.

We use the following three equations:

(3) $$r_{p,t} = \alpha_p + \beta_{p,t} r_{m,t} + \varepsilon_{p,t}$$

(4) $$(1 - \phi L)(\beta_{p,t} - \mu) = \boldsymbol{\lambda}' \mathbf{z}_{t-1} + \varepsilon_{\beta,t}$$

(5) $$r_{m,t} = \boldsymbol{\gamma}' \mathbf{z}_{t-1} + \varepsilon_{m,t} \,.$$

Equation (3) models excess portfolio returns over the risk-free rate at time *t*. In this equation α_p denotes the risk-adjusted abnormal return, i.e. Jensen's alpha; $\beta_{p,t}$ denotes the portfolio's

[§] This is the key assumption in our model: we develop a Bayesian asset pricing model in which betas are not merely deterministic functions of state variables. The Bayesian approach is now largely used in asset pricing literature, both to study the stock returns (Avramov, 2002; Cremers, 2002), and fund managers (Meligkotsidou and Vrontos, 2008; Giannikkis and Vrontos, 2011). Savona (2012) uses the framework developed on an earlier version of our paper (Amisano and Savona, 2008) to study hedge funds performance.

exposure to systematic risk which is assumed to be time varying, $r_{m,t}$ indicates the excess market return over the risk-free rate and $\varepsilon_{p,t}$ the unexpected portfolio return.

Equation (4) describes how beta varies in time. Here L denotes the lag operator, ϕ the persistence parameter on beta, μ the unconditional mean of beta, $\mathbf{z}_{t-1}$ the vector of conditioning variables at time $t-1$, $\boldsymbol{\gamma}$ the vector of sensitivities of beta with respect to conditioning variables and $\varepsilon_{\beta,t}$ the stochastic component to accommodate imperfect predictability in beta evolution[**]. This term can also be viewed as a noisy private signal about future market returns.

Note that our beta specification accommodates as special cases several patterns of time variation for beta. When $\boldsymbol{\lambda} = \mathbf{0}$, i.e. when the predictors $\mathbf{z}_{t-1}$ have no effect on beta, then we have the model proposed by Jostova and Philipov (2005). When $\phi \equiv 1$ the process has a unit root, with shocks in beta that persist indefinitely; when $\phi \equiv 0$ the process exhibits instantaneous mean reversion. Notice that, given stationarity of $\mathbf{z}_{t-1}$, stationarity of beta requires $|\phi| < 1$.

Equation (5) describes market excess returns over the risk-free rate as a linear function of the same predictors in (4), with $\boldsymbol{\gamma}$ denoting the vector of sensitivities and $\mathbf{z}_{t-1}$ the vector of predictors at time $t-1$, and $u_{m,t}$ is the unexpected market return at time t, hence accommodating imperfect predictors.

[**] In contrast, conditional asset pricing models as in Ferson, Schadt (1996) consider a predictive regression approach in which the linear combination of lagged predictors assumes that the true conditional expected beta is explained perfectly, i.e. without error, by observed predictors.

To close the model, we impose a structure on the innovations appearing in all equations. Indeed, it seems plausible to assume a positive-definite covariance matrix whose off-diagonal elements could arise as a result of a market timing ability as well as leverage effects on beta. Hence, the assumption is that the system innovations exhibit the following i.i.d. distribution:

(6)
$$\begin{bmatrix} \varepsilon_{p,t} \\ \varepsilon_{\beta,t} \\ \varepsilon_{m,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{\Sigma} \right)$$
$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_p^2 & \sigma_{p\beta} & \sigma_{pm} \\ \sigma_{p\beta} & \sigma_\beta^2 & \sigma_{\beta m} \\ \sigma_{pm} & \sigma_{\beta m} & \sigma_m^2 \end{bmatrix} = \begin{bmatrix} \sigma_p^2 & \rho_{p\beta}\sigma_p\sigma_\beta & \rho_{pm}\sigma_p\sigma_\beta \\ \rho_{p\beta}\sigma_p\sigma_\beta & \sigma_\beta^2 & \rho_{\beta m}\sigma_\beta\sigma_m \\ \rho_{pm}\sigma_p\sigma_\beta & \rho_{\beta m}\sigma_\beta\sigma_m & \sigma_m^2 \end{bmatrix}.$$

where $\sigma_i^2, \sigma_{ij}, \rho_{ij}, i = \{p, \beta, m\}, j = \{p, \beta, m\}$ denote respectively shock variances, covariances and correlation coefficients.

3. Estimation Approach

Model described by equations (3)-(6) is a state space model in which beta is treated as a latent variable. In particular, equation (3) is the measurement equation, equation (4) is the state equation, and equation (5) endogenizes the covariates appearing in the measurement and state equations. Equation (6) describes the statistical properties of the shocks affecting the system.

To estimate the model we use a state-space based Bayesian approach, treating model parameters as random variables. Bayesian inference (see for instance Geweke, 2005, Chapter 2) customarily starts by formulating a model and beliefs about model parameters and latent variables implied by the model in the form of a multivariate probability distribution, taking values on a given domain.

Data are analyzed through the lens of the model and inference is conducted by simulating the joint posterior distribution of parameters and latent variables. Data enter this process through the likelihood function, i.e. the density of observed data conditional on the model and on parameter values.

We denote by $\boldsymbol{\theta} = \left[\alpha_p, \phi, \mu, \boldsymbol{\gamma}^{'}, \boldsymbol{\lambda}^{'}, vech(\boldsymbol{\Sigma})^{'}\right]$ the vector containing all parameters and by $p\left(\boldsymbol{\beta}_{p,T}, \boldsymbol{\theta}\right)$ the joint prior distribution for parameters and latent variables (the betas). The prior distribution for the betas conditional on parameters $p\left(\boldsymbol{\beta}_{p,T}, \boldsymbol{\theta}\right)$ is given by equation (4), while the prior distribution of the parameters $p\left(\boldsymbol{\beta}_{p,T}, \boldsymbol{\theta}\right)$ must be explicitly provided by the researcher.

Obtaining the likelihood function requires integrating out the model's latent variables:[††]

(7)
$$
\begin{aligned}
&p\left(\mathbf{r}_{pT}, \mathbf{r}_{mT} \middle| \boldsymbol{\theta}\right) \equiv \int p\left(\mathbf{r}_{pT}, \mathbf{r}_{mT} \middle| \boldsymbol{\theta}, \boldsymbol{\beta}_{p,T}\right) p\left(\boldsymbol{\beta}_{p,T} \middle| \boldsymbol{\theta}\right) d\boldsymbol{\beta}_{p,T} \\
&\boldsymbol{\beta}_{p,T} = \left[\beta_{p,0}, \beta_{p,1}, \ldots, \beta_{p,T}\right]', \\
&\mathbf{r}_{pT} = \left[r_{p,1}, r_{p,2}, \ldots, r_{p,T}\right]', \\
&\mathbf{r}_{mT} = \left[r_{m,1}, r_{m,2}, \ldots, r_{m,T}\right]'.
\end{aligned}
$$

The joint posterior distribution of parameters and latent variables (the betas) is obtained by means of Bayes' theorem as follows:

(8)
$$
p\left(\boldsymbol{\beta}_{p,T}, \boldsymbol{\theta} \middle| \mathbf{r}_{pT}, \mathbf{r}_{mT}\right) \propto p\left(\boldsymbol{\beta}_{p,T}, \boldsymbol{\theta}\right) p\left(\mathbf{r}_{pT}, \mathbf{r}_{mT} \middle| \boldsymbol{\theta}, \beta_{p,t}\right).
$$

In this paper, following Kim and Nelson (1999, chapters 7 and 8), this posterior distribution is simulated using a Gibbs sampling-data augmentation procedure, a Markov Chain Monte Carlo

[††] In order to (slightly) simplify notation, here we leave dependence on the variables **z** completely implicit. This is perfectly legitimate since these variables are assumed exogenous.

(MCMC) technique that generates random samples from a given target distribution, namely the joint posterior distribution of the parameters $\boldsymbol{\theta}$ and the state variables given the observed returns $p\left(\boldsymbol{\beta}_{p,T},\boldsymbol{\theta}\middle|\mathbf{r}_{pT},\mathbf{r}_{mT}\right)$. The presence of endogenous regressors in the state and measurement equations requires some modifications to the standard MCMC approach for state space models described in Kim and Nelson (1999). The simulation strategy is detailed in Section B of the online appendix[‡‡].

3.1. Priors

Prior specification is certainly the most difficult and controversial aspect of Bayesian inference. In principle, prior distributions should allow the researcher to incorporate extra-sample information in a consistent manner. We construct our prior by exploiting what we can learn from a "training sample", i.e. a set of observations preceding those used in the empirical analysis. This is particularly easy for the regression parameters in the equation for market returns equation (5). Calling $\hat{\boldsymbol{\gamma}}_0$ and $\hat{\mathbf{S}}_{\gamma,0}$ the OLS estimate of $\boldsymbol{\gamma}$ and its estimated covariance matrix obtained by using only the observations belonging to the training sample (t = 1, 2, …, T_0), for $\boldsymbol{\gamma}$ we use a Gaussian prior centred on $\hat{\boldsymbol{\gamma}}_0$ and prior covariance matrix equal to $4\times\hat{\mathbf{S}}_{\gamma,0}$.

The training sample approach however is more complicated for the coefficients in the state equation, given the presence of latent variables. We therefore use the following two-step procedure:

[‡‡] The online appendix to this paper is available at the URL: `www.eco.unibs.it/~amisano/AS2013/AS_2013_appendix.pdf`.

1) On the training sample observations (t = 1, 2, …, T_0) we apply the Ferson and Schadt (1996) conditional performance approach and we estimate by OLS the following equation:

(9) $$r_{p,t} = \alpha_p + \beta_p(\mathbf{z}_{t-1}) r_{m,t} + e_{p,t}$$

(10) $$\beta_p(\mathbf{z}_{t-1}) = b_{0p} + \mathbf{b}'_p \mathbf{z}_{t-1} \ .$$

Here betas are deterministically projected on the conditioning variables $\mathbf{z}_{t-1}$.

2) The resulting estimates $\hat{\beta}_p(\mathbf{z}_{t-1}) = \hat{b}_{0p} + \hat{\mathbf{b}}'_p \mathbf{z}_{t-1}$ are used as observable counterparts of betas in equation (4), which is then estimated using the training sample observations (t = 1, 2, …, T_0). Denoting $\hat{\boldsymbol{\psi}}_0$ and $\hat{\mathbf{S}}_{\psi,0}$ the OLS estimate of $\boldsymbol{\psi}$ = [μ, ϕ, $\boldsymbol{\lambda}'$]$'$ and its estimated covariance matrix obtained by the training sample (t = 1, 2, …, T_0), for $\boldsymbol{\psi}$ we define a Gaussian prior centred on $\hat{\boldsymbol{\psi}}_0$ and prior covariance matrix equal to $\frac{1-R^2}{R^2} \times \hat{\mathbf{S}}_{\psi,0}$, where R^2 is the goodness of fit index in the pre-sample estimation of the equation for market returns. This scaling is done to reflect the importance of market returns conditional predictability in determining how precisely the sensitivity parameters $\boldsymbol{\lambda}$ can be estimated.

3) The same logic is used to calibrate the prior for α_p: we define a Gaussian prior centered on the OLS estimate of equation (9) based on the pre-sample and with prior variance equal to the estimated variance of the OLS estimate.

In order to specify a prior on the $\mathbf{\Sigma}$ covariance matrix of shock innovations, we use the OLS estimated residuals over the training sample $(\hat{\boldsymbol{\varepsilon}}_t, t = 1,2,...,T_0)$ to calibrate a Wishart prior for the inverse of $\mathbf{\Sigma}$:

$$\mathbf{\Sigma}^{-1} \sim Wishart(\underline{\mathbf{v}}, \underline{\mathbf{S}}), \qquad (11)$$
$$\underline{\mathbf{v}} = \frac{T_0}{4}, \underline{\mathbf{S}} = 4 \times \left(\sum_{t=1}^{T_0} \hat{\boldsymbol{\varepsilon}}_t \hat{\boldsymbol{\varepsilon}}_t^{'} \right)^{-1}.$$

In this way we centre the prior on values which are coherent with those observed in the training sample and we rescale degrees of freedom and scale parameters (by multiplying by 4) in order to emphasise prior uncertainty.

Summing up, the specification of our model and of the prior distributions used for estimation is guided by the central assumption that managers modulate systematic risk of their portfolios in part by observing how the benchmark returns are related to some predictors, and in part on the basis of their own information set which is unobservable to the econometrician. This is why the same set of covariates $\mathbf{z}_{t-1}$ enters both beta benchmark equations. By changing portfolio composition, i.e. modifying betas, managers take into account potential benefits deriving from market timing. In turn, market timing arises from benchmark predictability and from truly private information. This explains why the equation for beta postulates dependence on the covariates and the presence of a stochastic term, orthogonal to past predictors but potentially correlated with unexpected market returns. This correlation is the component of market timing that is not connected with the capacity to anticipate market returns and truly reflects private information.

3.2. Instruments for Beta Process

The specification of the set of variables **z** to be used as predictors is a crucial aspect of the specification of our model. The literature on predictability of equity returns using lagged values of economic and financial variables is extensive. Here we refer to the paper by Ait-Sahalia and Brandt (2001), on how to select and combine variables to best predict optimal portfolio weights of an investor who works with the following set of predictive variables:

1) the credit (or default) spread, computed as the yield difference between Moody's BAA and AAA-rated corporate bonds;
2) the log dividend-to-price ratio of the S&P index, computed as the annual dividend yield of the CRSP value-weighted stock index (other works used the sum of dividends paid on the S&P index over the past 12 months divided by the current level of the index);
3) S&P index trend (or momentum) variable computed as the difference between the log of the current S&P index level and the log of the average index level over the previous 12 months;
4) the term structure slope (or, simply, term spread), measured as the difference between a five-year and a one-month discount Treasury yield (other works use the yield difference between the ten and one-year government bonds).

As noted by Ait-Sahalia and Brandt (2001), the economic rationale of this selection is provided by Fama and French (1988, 1989), who show that the first three predictors capture cyclical time variations in excess stock and bond returns, and Keim and Stambaugh (1986), who use a variable similar to trend in order to predict returns. For our paper, such a choice seems theoretically and methodologically consistent, since Ait-Sahalia and Brandt (2001) have shown how to select and combine variables to optimally predict an investor's optimal portfolio weights.

Note that other studies often start by considering a larger set of predictors and, using some preliminary statistical analysis, they narrow down the set of included predictors to generally 4-6 factors. Based on Ferson and Qian (2004) we limit the number of instruments to 4 so as to have a parsimonious model, easy for estimation and interpretation of the results.

3.3. Posterior simulation of the model

As already mentioned, equations (3), (4), (5) and the set of assumptions (6) on the model shocks describe a Gaussian state space system. It is not a standard linear state space model, owing to the multiplicative interaction between $\beta_{p,t}$ and $r_{m,t}$ in equation (3), since $r_{m,t}$ is treated as an endogenous variable. Nevertheless, the joint posterior distribution of parameters and latent variables can be easily simulated by means of a simple and intuitive Gibbs sampling-data augmentation procedure. With this in mind, we can partition latent variables and parameters in the following five subsets:

1. the latent variables β_t, t = 1, 2, ... T;
2. α_p, the intercept parameter in equation (3) ;
3. the regression parameters in equation (4), namely $\mu, \phi, \boldsymbol{\lambda}$;
4. the regression parameters in equation (5), namely $\boldsymbol{\gamma}$;

5. the parameters in $\boldsymbol{\Sigma}$, i.e. the second moment structure of all shocks in the model.

Given the priors, each subset of random variables is simulated from its conditional posterior distribution. Details of these conditional distributions, and of how to simulate them, are given in Section A of the online appendix.

Simulation results are carried out by running 110,000 replications and discarding the first 10,000 to achieve convergence to the posterior distribution for each of the funds' portfolios analysed. The MCMC algorithm converged for all funds' portfolios.

4. Data: Mutual funds, Benchmark and Predictors

We use monthly returns of 5,337 open-ended US equity funds from January 1980 to December 2005, as supplied by the Centre for Research in Security Prices (CRSP) Survivor-Bias Free U.S. Mutual Fund Database. The time period was split into two intervals, the first from January 1980 to December 1989 and the second from January 1990 to December 2005. The first sub-sample is the "training sample" used for prior calibration along the lines described in Section 3.1. The second sub-sample, the "estimation sample" is used to generate the posterior distributions of parameters and latent variables. Mutual funds included in the sample have at least two years of data (48 monthly observations) in the period from January 1990 to December 2005. Monthly returns are calculated as total returns, therefore reflecting the reinvestment of dividends and capital gains.

We decided to work with equally weighted portfolios (EWPs) of mutual funds according to the ownership style category as provided by Standard and Poor's Style Name. The reason why we deal with EWPs is twofold. First, mutual fund benchmarks summarise the behaviour of homogeneous classes of funds in terms of style investing, thus giving a “general” view of each specific style, which is particularly useful in identifying common characteristics of funds. Second, given the high number of individual funds in the sample, it would have been computationally intensive to conduct the analysis at an individual funds level and, most likely results would have been extremely noisy.

One drawback of working with funds’ portfolios is that single managers' idiosyncratic information is averaged out, but operating at a style level allows to look at funds features which are homogeneous among funds being aggregated in the single portfolios.

The seventeen different EWPs are listed in Table 1.A detailed synthetic data description is contained in Section B of the online appendix.

[Table 1 to appear here]

5. Mutual Fund Performance and Beta Dynamics

Before inspecting mutual fund performance and beta dynamics, we firstly verify the predictability of the selected instruments by running equation (5), using the four demeaned and standardised predictors and controlling for heteroskedasticity and autocorrelation with the Newey-West (1987) covariance estimator. Unreported results (available upon request) show low R^2 for the two subsamples and for the whole sample of available data (never exceeding 3 per cent), which is consistent with recent empirical evidence (e.g., Campbell and Thompson, 2007). Given the structure of our prior distribution, low predictability entails diffuse priors on the parameters on the conditioning variables in equation (4). In other words, given the weak predictability, the potential benefits of market timing through predictability look extremely low and the prior reflects expectations that managers do not mechanically follow timing rules. Indeed, we expect λ and γ to differ in signs and magnitudes.

We then run the system (3)-(6) for each EWP with a prior elicited as described in Section 3.1.

From equation (3), note that α is constant and represents a Bayesian measure for each EWP's Jensen’s alpha. Since beta is a mean-reverting process affected by imperfect predictors and by stochastic shocks, , our model can deliver unbiased estimates of excess returns. in addition, beta dynamics gives direct information on market timing ability conditional on public information within a Bayesian context via correlations across shocks as described in equation (6). In other words, the model delivers a Bayesian Conditional Market Timing measure.

Table 2 reports the posterior means of parameters and Table 3 shows correlations of parameters posterior means across different EWPs. These correlations are intended to provide measures of interrelations of estimates across different portfolios.

Note that in computing correlations for shocks in the system we only refer to off-diagonal elements, since they give information on various angles of mutual fund performance; the off-diagonal elements we used are not covariances but correlations, computed by using the parameter in (6).

[Table 2 to appear here]

[Table 3 to appear here]

Jensen's alphas are positive and statistically significant for 12 out of 17 EWPs, implying that most funds deliver positive extra performance over the inspected period. Interestingly, in Table 3 we observe that correlation between alphas and $\rho_{p\beta}$ estimates is – 0.53 and significant at the 5% level. Given that $\rho_{p\beta}$ measures the leverage effect in portfolio returns, this result shows that, across portfolios, a lower leverage is associated to higher Jensen's alpha. To better understand this result we need to inspect the relationship between leverage effect and market timing since, as we will discuss in the next section, we obtain an estimated negative market timing. This means that, on the one hand, mutual fund managers react to benchmark returns shocks in an unexpected direction. On the other hand, the leverage effect seems to compensate this effect. Moreover, the correlation between shocks in mutual fund returns and shocks in benchmark returns, ρ_{pm}, is negatively related to the leverage effect and positively related to the market timing: Table 3 shows that these correlation coefficients, both statistically significant, are respectively – 0.65 and 0.57. The latter correlation suggests that, in spite of the negative market timing, shocks in benchmark returns and shocks in portfolio returns are negatively correlated. This further suggests a hedging effect played by innovations in portfolio returns, which tend to offset positive and negative benchmark surprises. In other terms, fund managers seem to be more focused on long-run hedging strategies rather than pure and aggressive market timing strategies. Indeed, such investment strategy seems profitable, as the Jensen's alpha is significantly positive for most of the EWPs.

Inspection of the parameters describing beta dynamics gives interesting insights on how managers modify their risk exposure over time. Let us start by considering the persistence parameter ϕ. The estimated values indicate high average persistence in beta variation, though some differences occur for specific EWP depending on predictors. The mean coefficient is 0.45 and takes on values between – 0.09 and 0.81. Interestingly, significant persistence coefficient tends to be high, indicating significant speed in mean-reversion and so high beta volatility.

Another interesting finding contained in Table 3 is that the correlation between the long long-run mean and persistence parameters for beta is significant and negative, with a value of– 0.95. This suggests that dynamic funds with significant risk exposure variation have in general low beta on average, since high persistence reflects high unconditional beta volatility.

Results in Tables 2 and 3 regarding beta sensitivities with respect to predictors reveal that this set of variables matter, at least for specific fund category, and that funds differ significantly in terms of instrument-based rules in beta variations. Assuming that managers look at predictors in estimating expected benchmark returns before choosing the right risk exposure, the term spread and trend seem to be the most important predictors with the higher absolute average coefficients. Indeed, we can calculate averages of absolute coefficient values as follows: 0.0275, 0.0254, 0.0290, 0.0439 for the first, second, third and fourth predictors.

Considering the statistical significance of the coefficients, we note that we have at least a significant predictor for 12 out of 17 EWPs. For each individual predictor, the default spread is significant for six style categories while for dividend yield is significant only for three EWPs. Term spread and trend are significant in nine and ten cases, respectively.

We also note that the term spread coefficient is positive for 5 categories, while for the remaining four EWPs the coefficient is negative. The term spread is known to move with the business cycle, making it a natural predictor of equity and bond returns[§§]. The positive coefficient we obtain in our analysis is thus economically reasonable, while a negative sensitivity might be associated with funds focusing on specialized categories with returns weakly correlated with the overall U.S. equity market. In fact, the funds that exhibit negative term spread coefficients are the Energy InfoTech Materials sectors.

Regarding the Trend predictor, nine out of ten significant coefficients have a positive sign, indicating therefore a general tendency to behave as momentum funds. The positive coefficients estimated for default spread (five out of six significant coefficients) and dividend yield (two out of three significant coefficients) are consistent with the fact that the two variables track variation in expected returns as they are largely measures of business conditions (Fama and French, 1989).

The relationship between γ and λ coefficients, which can be seen by inspecting correlations between each element of γ with the corresponding one of $\lambda$is also of particular interest. Let us recall that the λ coefficients define predictability of the benchmark returns on the basis of predictors, while the γ coefficients denote the dependence of betas on predictors. The correlations reported in Table 5 indicate that the linear relationships among these two sets of coefficients are virtually non-existent: managers do not care about benchmark sensitivities in choosing their instrument exposure. A first and obvious reason may be related to the scant predictability of

[§§] Chen (1991) and Fama and French (1989) find a positive relation between the yield spread and the future equity and bond returns.

benchmark returns using instruments. Furthermore, it cannot be excluded that γ and λ may be related in a more complex, non-linear relationship. This possibility will be investigated in our future research.

6. Betas and Market Timing

Within our model's framework, a manager is market timer if $\rho_{\beta m}$ is positive. As in Becker *et al.* (1999), we distinguish timing ability that merely reflects publicly available information as captured by the set of instrumental variables, from conditional market timing based on better information. But, unlike these authors, we also model imperfect predictability and a stochastic component in the process for systematic risk. Table 2 reports posterior estimates of the correlations among shocks in the system, giving information on both Bayesian conditional market timing ability measured by $\rho_{\beta m}$ and leverage effect measured by $\rho_{\beta p}$. Furthermore, Table 3 reports the correlation between portfolio return innovations and benchmark innovations, i.e. ρ_{pm}. Note that portfolio innovations are functions of beta innovations, and so ρ_{pm} should depend on the correlation between beta and benchmark innovations.

Let us start with $\rho_{\beta m}$, which signals conditional market timing ability. Table 2 shows that no mutual fund category was a significant market timer over the 1990-2005 period. Indeed, no correlation appears to be positive and statistically significant. We note however that only the Info Tech Sector exhibits a positive correlation of 0.12 which is almost significant at 0.1 level (p-value equal to 0.105) and may lead us to consider the Info Tech Sector as a "persistent" market timer over the time period inspected. Interestingly, this is the unique fund category for which we detect some conditional market timing. This result is consistent with the existing literature on market timing activity of mutual funds. In fact, Elton, *et al*. (2012) , using conditional sensitivities obtained by extending Ferson and Schadt (1996), find slight evidence of negative market timing. Differently from their results, which are not statistically significant, our findings denote instead that 13 out 17 EWPs exhibit significant negative market timing. As discussed in the previous section, a clear interpretation of our results requires a joint interpretation of market timing, leverage effect, and Jensen's alpha.

First, from Table 3 we note that the leverage effect, as measured by $\rho_{p\beta}$, is significant for four EWPs with two positive coefficients and two negative coefficients. On the other hand, ρ_{pm} is significant for ten EWPs, most of them with negative values. These results suggest, therefore, that what really matters is the interaction between leverage and market timing. Since on average we have negative conditional market timing, the negative combined effect between leverage and market timing effect may be, in a sense, good for mutual funds. In fact, anomalous negative timing effects (as measured by negative $\rho_{\beta m}$) would be indeed mitigated by innovations in portfolio returns leading to positive Jensen's alphas.

This is also what we find by computing correlations among the parameters. In fact, Table 3 shows that $\mathrm{corr}\lfloor\rho_{p\beta}, \rho_{pm}\rfloor$ is significantly negative (– 0.65). Interestingly, in the same table we note that the correlation between the leverage and Jensen's alpha is significantly negative (–

0.52). So, it seems that the negative leverage effect not only compensates the negative market timing, but also leads to an increment in the extra performance of mutual funds, i.e. selectivity.

There is also another important point to consider when interpreting beta dynamics and their impacts on portfolio performance. If changes in systematic risk exposure are connected to conditioning variables and unobservable shocks, a very important question is how these components contribute in explaining the variation in beta, and in turn, the mutual fund performance. To this end, in the next section we take up a variance decomposition of the betas.

7. Beta Decomposition

Our interest in decomposing the beta variance and thus assessing the relative contribution of persistence and conditioning variables variability and shocks in explaining beta dynamics, is strongly related to Mamaysky *et al.* (2008). As discussed in the introduction, they did not find any significance in adding Treasury bill and dividend yield on the CRSP equally-weighted index in their state space model. The estimated coefficients were statistically equal to zero, thus leading them to conclude that very few funds use macroeconomic variables.

This is precisely our main concern: how much do predictors impact on beta dynamics? More broadly, we want to measure the relative importance of different sources in explaining the observed variance of betas at all horizons..

To do so, we use the state space estimates for the betas and market returns and we augment the model by adding a VAR specification for predictors. This extended system is indeed a VAR model and we use the structural VAR approach of Amisano and Giannini (1997) to decompose the instantaneous correlations across shocks.

The model is based on the following two assumptions:

1) the unexpected part of beta is allowed to depend on the unexpected component of the market benchmark;
2) the unexpected market benchmark return is allowed to depend simultaneously on shocks of the predictors.

These two reasonable assumptions allow us to identify uncorrelated shocks affecting beta and in particular:

a) an idiosyncratic shock on beta (the so-called own shock);
b) a shock on the benchmark;
c) a set of shocks affecting the predictors.

Analytically, the VAR specification is as follows:

$$(12)\quad \begin{bmatrix} 1 & 0 & a_{\beta m} \\ 0 & P^{zz} & 0 \\ 0 & p^{zm} & p^{mm} \end{bmatrix} \begin{bmatrix} \beta_t - \mu \\ z_t \\ r_{m,t} \end{bmatrix} = \begin{bmatrix} \phi & \lambda' & 0 \\ 0 & A & 0 \\ 0 & \gamma' & 0 \end{bmatrix} \begin{bmatrix} \beta_{t-1} - \mu \\ z_{t-1} \\ r_{m,t-1} \end{bmatrix} + \begin{bmatrix} f_{\beta\beta} & 0 & 0 \\ 0 & I_k & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_{\beta,t} \\ e_{z,t} \\ e_{m,t} \end{bmatrix}$$

where $\begin{bmatrix} e_{\beta,t} \\ e_{z,t} \\ e_{m,t} \end{bmatrix}$ is a vector of orthogonal shocks with unit variances and $P^{-1} = \begin{bmatrix} P^{zz} & 0 \\ p^{zm} & p^{mm} \end{bmatrix}$ is the inverse of the Cholesky factor of the one step ahead covariance matrix of the vector $\begin{bmatrix} z_t \\ r_{m,t} \end{bmatrix}$. Given estimates of the state space model (1) - (3) and the estimate of a VAR model for the predictors, it is possible to back out the coefficients of the structural representation (12) and to compute variance decompositions for all betas. Table 5 reports results of variance decompositions in terms of relative importance attributed to each main driver of beta variability at 1 and 6 months horizon. On average, predictors really matter in explaining how the systematic risk exposure changes over time. This strongly contrasts with the findings of Mamaysky *et al.* (2008). At the one month horizon predictors account for 16%of the total beta variability, while at the six month horizon the quota is 65%. Idiosyncratic beta shocks have major role in the short-run, by accounting for 4% on average, hence suggesting that managers strongly use their own private signals to reallocate their portfolios. However, in the medium-run, they account for approximately one third of the total variation in systematic risk variation, while predictors significantly increase their quota. Benchmark innovations play instead a minor role both in the short- and in the medium-run, showing quotas of less than 1%.

These results suggest, therefore, a tendency of mutual funds to converge towards instrument-based investment strategies. Furthermore, the negligible role assumed by benchmark surprises is consistent with the insignificant market timing ability documented by our results..

8. Conclusion

How does a manager use predictors in changing her/his portfolio structure over time? This is what we analyse in this paper. To do this we derived a new model which combines a stochastic component, and a systematic component in the time variation pattern. To this end we use a predictive system that jointly considers portfolio excess returns, a time-varying beta and the benchmark excess returns. Predictors are assumed to be imperfect and innovations in the system are instantaneously correlated

We use this model to inspect how managers use predictors in changing their investment strategies over time. In addition, our model delivers a measure for conditional market timing which is different from that obtained from traditional conditional asset pricing models, since we accommodate imperfect predictors and correlation across innovations.

Our empirical study is conducted on 17 equally weighted sectoral portfolios over the 1980-2005 period, and shows that instruments impact significantly on beta dynamics, but managers do not care about benchmark sensitivities towards predictors in choosing their instrument exposure. Intuitively, this result is due to the modest benchmark forecasting abilities of these variables. Persistence in beta is significant although we note strong differences across fund categories. Interestingly, betas' long-run means and persistences are negatively correlated.

In accordance with the existing literature on market timing, we do not find evidence of market timing ability. On the other hand, we do find a significant leverage effect. When the market timing is anomalously negative, the negative leverage effect offsets the negative timing effect. As a consequence, most of the mutual fund categories exhibit significantly positive Jensen's alpha. Using a structural VAR approach to decompose the beta variance, we find that mutual funds implement instrument-based investment strategies showing scarce attention towards benchmark surprises, which is consistent with anomalous negative market timing ability.

Kacperczyk *et al*. (2011) recently offered new interesting insights on the time variability of market timing and stock picking. By exploring the performance of actively managed open-end

U.S. equity mutual funds from January 1980 until December 2005, the authors show significant stock-picking skills in booms and market-timing skills in recessions when conditioning on the state of the business cycle. Our model and methodology can be extended to analyze time varying market timing and stock picking and we plan to do so in our future research

.

References

Admati, A., and S. Ross, 1985, "Measuring Investment Performance in a Rational Expectations Equilibrium Model," *Journal of Business*, 58, 1–26.

Admati, A., S. Bhattacharya, P. Pfleiderer, and S. Ross, 1986, "On Timing and Selectivity," *Journal of Finance*, 41, 715–730.

Ait-Sahalia, Y., and M. Brandt, 2001, "Variable Selection for Portfolio Choice," *Journal of Finance*, 56, 1297–1351.

Amisano, G. and C. Giannini, 1997, "*Topics in structural VAR econometrics*", Springer, New York.

Amisano, G. and R. Savona, 2008, "On the dynamics of equity mutual funds performances: a Bayesian perspective", *ECB Working Papers Series* n. 831.

Avramov, D., 2002, "Stock return predictability and model uncertainty," *Journal of Financial Economics*, 64, 423–458.

Becker, C. W. Ferson, D. Myers, and M. Schill, 1999, "Conditional Market Timing with Benchmark Investors," *Journal of Financial Economics*, 52, 119–148.

Campbell, J., and S. Thompson, 2007, "Predicting Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average?," *Review of Financial Studies*, forthcoming.

Carter, C., and R. Kohn, 1994, "On Gibbs Sampling for State Space Models," *Biometrika*, 81, 541–553.

Chen, N., 1991, "Financial Investment Opportunities and the Macroeconomy," *Journal of Finance*, 46, 529–554.

Chen, Y., W. Ferson, and H. Peters, 2010, “Measuring the Timing Ability of Fixed Income Mutual Funds,” *Journal of Financial Economics*, 98, 72-89.

Connor, G., 1997, “Sensible Return Forecasting for Portfolio Management,” *Financial Analyst Journal*, 53, 44–51.

Cremers, M., 2002, “Stock return predictability: a Bayesian model selection perspective,” *Review of Financial Studies*, 15, 1223–1249.

Elton E., M. Gruber and C. Blake, 2012, “An Examination of Mutual Fund Timing Ability Using Monthly Holdings Data”, *Review of Finance*, 16, 619–645.

Fama, E., and K. French, 1988, “Dividend Yields and Expected Stock Returns,” *Journal of Financial Economics*, 22, 3–25.

Fama, E., and K. French, 1989, “Business Conditions and Expected Returns on Stocks and Bonds,” *Journal of Financial Economics*, 25, 23–49.

Ferson, W. and M. Qian, 2004, *Conditional Performance Evaluation Revisited*, Research Foundation Monograph of the CFA Institute, ISBN 0-943205-69-7, 84 pages.

Ferson, W., and R. Schadt, 1996, “Measuring Fund Strategy and Performance in Changing Economic Conditions,” *Journal of Finance*, 51, 425–462.

Geweke, J., 2005, *Contemporary Bayesian Econometrics*, Wiley, New York.

Giannikis, D., Vrontos, S., 2011, “A Bayesian approach to detecting nonlinear risk exposures in hedge fund strategies,” *Journal of Banking and Finance*, 35, 1399–1414.

Glosten, L., and R. Jagannathan, 1994, “A Contingent Claims Approach to Performance Evaluation,” *Journal of Empirical Finance*, 1, 133–166.

Grinblatt, M., and S. Titman, 1989, “Portfolio Performance Evaluation: Old Issues and New Insights,” *Review of Financial Studies*, 2, 396–422.

Henriksson, R, and R. Merton, 1981, “On Market Timing and Investment Performance II: Statistical Procedures for Evaluating Forecasting Skills,” *Journal of Business*, 54, 513–534.

Johannes, M., and N. Polson, 2007, "MCMC Methods for Continuous-Time Financial Econometrics," in (eds. Ait-Sahalia, Y.) *Handbook of Financial Econometrics*, Elsevier, forthcoming.

Jostova, G., and A. Philipov, 2005, "Bayesian Analysis of Stochastic Betas," *Journal of Financial and Quantitative Analysis*, 40, 747–778

Kacperczyk, M., Van Nieuwerburgh, S., Veldkamp, L., 2011, "Time-Varying Fund Manager Skill," NBER Working Paper No. 17615.

Keim, D., and R. Stambaugh, 1986, "Predicting Returns in the Stock and Bond Markets," *Journal of Financial Economics*, 17, 357–390.

Kim, C.J, and C. R. Nelson (1999), *State space models with regime switching*, MIT Press, Cambridge, Massachusetts.

Meligkotsidou, L., Vrontos, I., 2008, "Detecting structural breaks and identifying risk factors in hedge fund returns: a Bayesian approach," *Journal of Banking and Finance*, 32, 2471–2481.

Merton, R., 1981, "On Market Timing and Investment Performance. I. An Equilibrium Theory of Value for Market Forecasts," *Journal of Business*, 54, 363–406.

Newey, W., and K. West, 1987, "Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," Econometrica, 55, 703–708.

Pastor, L, and R. Stambaugh, 2009, "Predictive Systems: Living with Imperfect Predictors," *Journal of Finance*, 64, 1583–1628.

a

Rubinstein, M., 1973, "A Comparative Statics Analysis of Risk Premiums," *Journal of Business* 46, 605–615.

Mamaysky, H., Spiegel, M., and H. Zhang, 2008, "Estimating the Dynamics of Mutual Fund Alphas and Betas," *Review of Financial Studies*, 21, 223–264.

Savona, R., 2012, "Risk and Beta Anatomy in the Hedge Fund Industry," *European Journal of Finance*, forthcoming.

Treynor, J., and K. Mazuy, 1996, “Can Mutual Funds Outguess the Market?,” *Harvard Business Review*, 44, 131–136.

Table 1: Equally weighted portfolios definitions

All Cap Growth	Large Cap Blend	Mid Cap Value
All Cap Value	Large Cap Growth	Small Cap Blend
Energy Sector	Large Cap Value	Small Cap Growth
Financials Sector	Materials Sector	Small Cap Value
Healthcare Sector	Mid Cap Blend	Utilities Sector
InfoTech Sector	Mid Cap Growth	

Table 2: Posterior means of parameters

	$\rho_{p\beta}$	ρ_{pm}	$\rho_{\beta m}$	α	ϕ	μ	γ_1 Def. spread	γ_2 Div. yield	γ_3 Trend	γ_4 Term spread
All Cap Growth	-0.0033	-0.0755	-0.3176***	0.0016	0.7127***	0.3419***	0.0054	0.0141	0.0124**	0.0048
All Cap Value	-0.0715	0.0883	-0.1187	0.0023***	0.6973***	0.2578	0.0023	0.0198	0.0053	0.0119
Energy Sector	0.1079	-0.2289***	-0.1834***	0.0028	0.2464	0.7458***	0.0619	-0.0224	0.0511**	-0.1609**
Financials Sector	-0.2344***	0.2613***	-0.1925***	0.0051***	0.0621	0.7344***	-0.0467***	0.0165	0.0204***	0.1358***
Healthcare Sector	0.1979***	-0.2025***	-0.1506***	0.0024	0.3769	0.6685***	-0.0384	0.033	0.0676**	0.0336***
InfoTech Sector	0.0623	0.3141***	0.1173	0.0019	-0.0874	1.2791***	0.0715***	-0.053***	-0.0182***	-0.0397***
Large Cap Blend	0.0563	-0.1698***	-0.4461***	0.0014***	0.752***	0.2324***	0.0036***	0.002	0.003**	0.0038**
Large Cap Growth	-0.0162	0.0819	-0.0444	0.0017***	0.7583***	0.2504	0.0029	0.006	0.0048	0.0023
Large Cap Value	-0.1699***	0.0779	-0.274***	0.0027***	0.2061	0.6733***	-0.0172	0.03**	0.0132**	0.0274***
Materials Sector	0.0717	-0.2021***	-0.2463***	0.0029	0.3577	0.5223***	0.1352**	0.0501	0.1501**	-0.1602***
Mid Cap Blend	-0.0704	0.0363	-0.3832***	0.0036***	0.3838	0.5167	-0.0028	-0.0315	0.0079	-0.0494
Mid Cap Growth	0.015	0.0975	-0.0032	0.0025**	0.0497	1.03***	-0.0112	0.0278**	0.017**	0.0433***
Mid Cap Value	0.055	-0.3211***	-0.2171***	0.0026***	0.8113***	0.1887***	0.0087***	0.013	0.0048	0.0053
Small Cap Blend	0.0469	-0.19***	-0.3936***	0.0026**	0.5341***	0.5042***	-0.0113	0.026	0.0376	0.0284***
Small Cap Growth	0.0048	0.091	-0.203***	0.003**	0.3155	0.7447	-0.0295	0.0396	0.0331	0.0258
Small Cap Value	0.1267**	-0.3273***	-0.4359***	0.0026**	0.7662***	0.2467***	0.0149**	0.0219	0.018**	0.0094
Utilities Sector	-0.034	0.203***	-0.221***	0.0039***	0.6721***	0.1313	-0.0038	-0.0248	-0.0289	0.0042
Mean	0.0085	-0.0274	-0.2185	0.0027	0.4480	0.5335	0.0086	0.0099	0.0235	-0.0043
Min	-0.2344	-0.3273	-0.4462	0.0014	-0.0875	0.1314	-0.0467	-0.0530	-0.0289	-0.1609
Max	0.1980	0.3141	0.1174	0.0052	0.8114	1.2792	0.1353	0.0502	0.1502	0.1358
StdDev	0.1057	0.2016	0.1535	0.0009	0.2894	0.3180	0.0444	0.0278	0.0399	0.0704

Note: the table reports posterior mean estimates of parameters obtained by MCMC posterior simulation of system (3)-(6) for each EWP. The table reports directly shock correlations. The superscript *, **, *** denote significance at 0.1, 0.5, and 0.01 levels respectively using posterior distribution estimated quantiles.

Table 3: Parameter Correlations

	$\rho_{p\beta}$	ρ_{pm}	$\rho_{\beta m}$	α	ϕ	μ	γ_1	γ_2	γ_3	γ_4
ρ_{pm}	**-0.6538**									
$\rho_{\beta m}$	-0.0143	**0.5677**								
α	**-0.5247**	0.3278	-0.0711							
ϕ	0.2124	**-0.5123**	**-0.5122**	-0.3524						
μ	-0.0017	0.4265	**0.5847**	0.0949	**-0.9472**					
γ_1	0.3696	-0.2208	0.1405	-0.2407	-0.0940	0.1203				
γ_2	0.0202	-0.3617	-0.2237	-0.0291	0.1375	-0.1573	-0.1727			
γ_3	0.3394	-0.4676	-0.1250	0.0490	-0.1641	0.1160	**0.4892**	**0.5839**		
γ_4	-0.4563	0.3577	0.0049	0.2344	0.0114	-0.0325	**-0.8638**	0.2437	**-0.4872**	
λ_1	**0.5665**	**-0.7359**	-0.4831	-0.1859	**0.6032**	**-0.5411**	0.2430	0.1406	0.1507	-0.2730
λ_2	-0.4549	**0.6370**	0.1334	0.0653	-0.2633	0.1843	-0.4130	-0.0346	-0.1953	0.3231
λ_3	0.3439	**-0.5701**	**-0.5314**	**-0.5723**	0.3820	-0.1989	-0.1951	0.1311	-0.1437	0.1054
λ_4	0.3445	**-0.5503**	-0.3558	-0.2295	0.3957	-0.3294	**0.5169**	-0.0010	0.0826	-0.4427

Note: the table reports correlation across posterior mean of parameters across the different EWPs. Numbers in boldface are significant at 0.05 level.

Table 4: Variance Decomposition of Betas

Horizon	1 month			6 month		
	own	benchmark	others	own	benchmark	others
All Cap Growth	81.40	0.70	17.90	64.10	0.61	35.29
All Cap Value	86.18	0.52	13.30	41.02	0.26	58.72
Energy Sector	86.48	0.55	12.97	23.91	0.19	75.90
Financials Sector	86.58	0.49	12.94	19.88	0.13	79.99
Healthcare Sector	87.40	0.44	12.16	22.29	0.15	77.57
InfoTech Sector	86.70	0.52	12.79	12.06	0.09	87.86
Large Cap Blend	74.96	0.96	24.08	62.06	0.85	37.09
Large Cap Growth	87.51	0.48	12.01	48.27	0.30	51.43
Large Cap Value	82.63	0.67	16.71	21.37	0.22	78.42
Materials Sector	84.80	0.66	14.54	24.13	0.24	75.63
Mid Cap Blend	78.48	0.82	20.70	33.64	0.44	65.91
Mid Cap Growth	87.86	0.46	11.67	19.70	0.12	80.17
Mid Cap Value	83.91	0.75	15.35	45.67	0.45	53.88
Small Cap Blend	77.49	0.93	21.58	33.64	0.50	65.86
Small Cap Growth	84.73	0.58	14.69	22.13	0.18	77.68
Small Cap Value	75.09	1.15	23.75	52.31	0.93	46.76
Utilities Sector	84.69	0.62	14.69	43.58	0.36	56.06
Mean	**83.35**	**0.66**	**15.99**	**34.69**	**0.35**	**64.95**
min	**74.96**	**0.44**	**11.67**	**12.06**	**0.09**	**35.29**
max	**87.86**	**1.15**	**24.08**	**64.10**	**0.93**	**87.86**
Std. deviation	**4.33**	**0.20**	**4.14**	**15.86**	**0.25**	**16.07**

Note: decomposition described in Section 7.

Appendix A: data description tables and figures

Table A: Descriptive Statistics of EWPs

	Mean	Median	Max	Min	StdDev	Skew	Kurtosis	ρ_1
Pre-sample: from 1980/01 to 1989/12								
All Cap Growth	0.0126	0.0143	0.1380	-0.2388	0.0587	-0.6996	5.2100	0.1230
All Cap Value	0.0118	0.0111	0.1155	-0.1963	0.0408	-0.8411	7.8104	0.1081
Energy Sector	0.0123	0.0142	0.1509	-0.2779	0.0574	-1.1655	8.3371	0.0421
Financials Sector	0.0134	0.0132	0.1012	-0.2138	0.0445	-1.0855	7.4413	0.2292
Healthcare Sector	0.0125	0.0151	0.1253	-0.2531	0.0531	-0.9797	7.2769	0.0964
InfoTech Sector	0.0111	0.0111	0.1910	-0.2822	0.0647	-0.4535	5.9564	0.0526
Large Cap Blend	0.0129	0.0163	0.1127	-0.2073	0.0442	-0.9075	7.2705	0.0874
Large Cap Growth	0.0142	0.0150	0.1344	-0.2323	0.0503	-0.8656	6.9542	0.1008
Large Cap Value	0.0139	0.0134	0.1062	-0.1879	0.0400	-0.9645	7.5575	0.0987
Materials Sector	0.0162	0.0055	0.2834	-0.2907	0.1021	-0.0666	3.5830	0.0148
Mid Cap Blend	0.0123	0.0140	0.1108	-0.2253	0.0435	-1.5491	10.1126	0.0891
Mid Cap Growth	0.0141	0.0166	0.1500	-0.2603	0.0537	-0.9285	7.6786	0.1058
Mid Cap Value	0.0133	0.0141	0.1436	-0.2113	0.0492	-0.5303	6.1128	0.0754
Small Cap Blend	0.0138	0.0180	0.1613	-0.2688	0.0553	-1.0177	7.7613	0.1414
Small Cap Growth	0.0144	0.0182	0.1367	-0.2695	0.0558	-1.0354	7.6123	0.1586
Small Cap Value	0.0125	0.0187	0.1483	-0.2336	0.0548	-0.9434	6.6086	0.2072
Utilities Sector	0.0145	0.0150	0.1410	-0.0909	0.0354	0.1954	4.6455	0.0990
Estimation Sample: from 1990/01 to 2005/12								
All Cap Growth	0.0104	0.0147	0.1796	-0.1864	0.0543	-0.2687	3.7526	0.0865
All Cap Value	0.0109	0.0138	0.0884	-0.1632	0.0385	-0.6928	4.7379	0.1136
Energy Sector	0.0100	0.0083	0.1859	-0.2060	0.0569	0.1933	4.5387	-0.0239
Financials Sector	0.0135	0.0206	0.1448	-0.2103	0.0475	-0.5703	5.0400	0.0780
Healthcare Sector	0.0128	0.0140	0.2247	-0.1660	0.0523	0.2579	4.7753	0.0607
InfoTech Sector	0.0145	0.0176	0.2792	-0.2772	0.0881	-0.2265	3.7749	0.0766
Large Cap Blend	0.0083	0.0121	0.1060	-0.1403	0.0380	-0.5143	3.8378	-0.0067
Large Cap Growth	0.0086	0.0111	0.1219	-0.1581	0.0462	-0.4803	3.6214	0.0374
Large Cap Value	0.0088	0.0123	0.1041	-0.1411	0.0370	-0.5236	4.2825	0.0118
Materials Sector	0.0075	0.0099	0.3430	-0.2062	0.0736	0.2808	4.7143	-0.0602
Mid Cap Blend	0.0108	0.0141	0.0971	-0.1759	0.0417	-0.6926	4.5624	0.0939
Mid Cap Growth	0.0106	0.0138	0.2129	-0.2001	0.0586	-0.2393	4.2383	0.0775
Mid Cap Value	0.0107	0.0141	0.1060	-0.1638	0.0384	-0.7119	5.1017	0.1123
Small Cap Blend	0.0103	0.0145	0.1261	-0.1899	0.0472	-0.6104	4.2442	0.1109
Small Cap Growth	0.0111	0.0172	0.2422	-0.2088	0.0637	-0.1723	4.0304	0.1008
Small Cap Value	0.0109	0.0181	0.0969	-0.1781	0.0422	-0.8028	4.8689	0.1945
Utilities Sector	0.0077	0.0079	0.0865	-0.1105	0.0327	-0.4765	3.7264	0.1074

Note: the table contains monthly descriptive statistics of equally weighted style matched portfolios provided by Standard and Poor's Style Name 5,337 of open-ended US equity mutual funds over the periods from January 1980 to December 1989 and from January 1990 to December 2005.

Table B: Returns and Predictors

Panel A: Descriptive Statistics

	Mean	Median	Max	Min	StdDev	Skew	Kurtosis	ρ_1
Pre-sample: from 1979/12 to 1989/11								
Default premium	0.0149	0.0140	0.0266	0.0076	0.0045	0.7744	2.8355	0.8758
Log dividend/price	1.4326	1.4573	1.8888	1.0039	0.2142	-0.0284	2.0655	0.9688
Momentum	0.0602	0.0708	0.2451	-0.2207	0.0951	-0.4853	2.5820	0.8585
Term Premium	0.0086	0.0130	0.0254	-0.0316	0.0119	-1.1313	3.8114	0.8904
Estimation Sample: from 1989/12 to 2005/11								
Default premium	0.0083	0.0079	0.0141	0.0053	0.0021	0.9613	3.1976	0.9358
Log dividend/price	0.6931	0.5958	1.3814	0.0831	0.3535	0.0896	1.8260	0.9827
Momentum	0.0420	0.0506	0.2352	-0.2636	0.0884	-0.8523	3.5918	0.8848
Term Premium	0.0143	0.0114	0.0331	-0.0049	0.0105	0.1943	1.6827	0.9800

Panel B: Correlations

Pre-sample: from 1979/12 to 1989/11

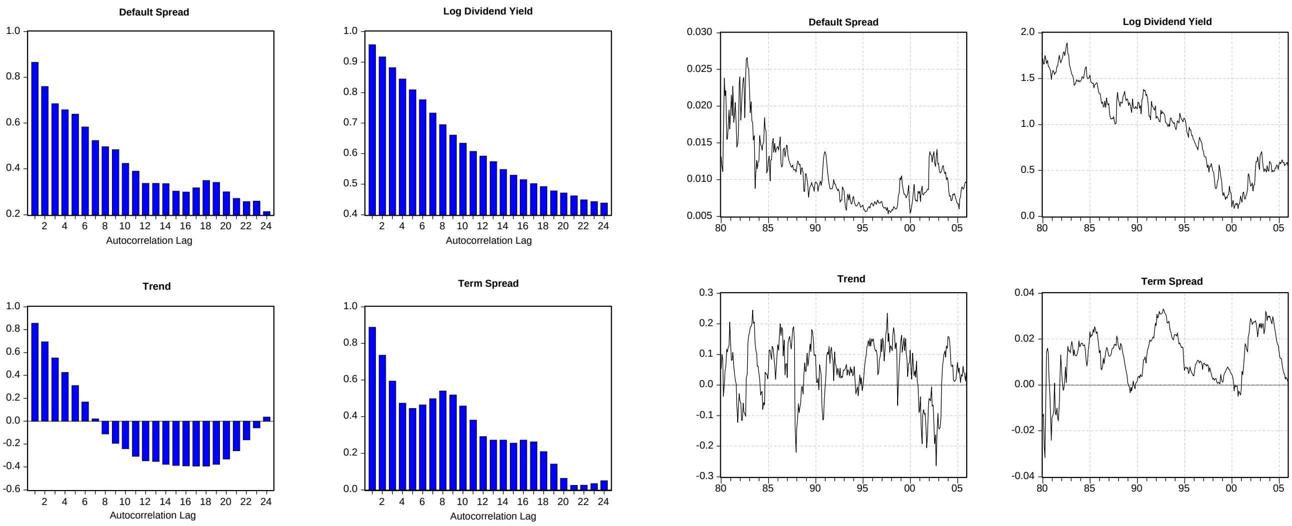
	Default premium	Log dividend/price	Momentum	Term Premium	r_m	$(r_m)^2$	$I_{rm>0}(r_m)$	$I_{rm<0}(r_m)$
Default premium	1.0000	0.7166	-0.1390	-0.1053	0.0728	0.0067	0.0998	0.0239
Log dividend/price		1.0000	-0.3941	-0.2900	0.0415	-0.0890	0.0255	0.0428
Momentum			1.0000	0.0188	-0.0485	-0.0221	-0.1033	0.0184
Term Premium				1.0000	0.1276	-0.0112	0.0625	0.1468

Estimation Sample: from 1989/12 to 2005/11

	Default premium	Log dividend/price	Momentum	Term Premium	r_m	$(r_m)^2$	$I_{rm>0}(r_m)$	$I_{rm<0}(r_m)$
Default premium	1.0000	0.0422	-0.4797	0.3794	-0.0276	0.1037	0.0388	-0.0822
Log dividend/price		1.0000	0.0176	0.3136	0.0956	-0.1434	-0.0106	0.1677
Momentum			1.0000	-0.2546	0.0180	-0.2121	-0.0917	0.1163
Term Premium				1.0000	-0.0210	-0.1398	-0.1154	0.0744

Note: Panel A contains monthly descriptive statistics for the four predictors used in this study: the default spread, , the log dividend-to-price ratio of the S&P index, the S&P index momentum variableand the term spread. Panel B shows correlations of the predictors with: the predictors, the excess stock return r_m its square $(r_m)^2$ and the Henriksson-Merton piece-wise term for upward and downward excess stock return, $I_{rm>0}(r_m)$ and $I_{rm<0}(r_m)$, namely the indicator variable for positive and negative excess stock return, $I_{rm>0}$ and $I_{rm<0}$, multiplied by the excess stock return (r_m).

Figure A: Predictors



This figure shows autocorrelograms and time-series plots for the default spread, the log-dividend-to-price ratio of the S&P index, the S&P index trend variable, and the trend variable. The data cover the period from January 1980 through December 2005 and refer to monthly observations.

Appendix B: conditional posterior distributions

B.1 Posterior simulation of the betas

Conditioning on all parameters and all the data on the benchmark $\left(r_{mt}, t = 1, \ldots, T\right)$ amounts to condition on the socks on market returns ε_{mt} and therefore considering the conditional distribution:

(B.1)

$$\left(\begin{matrix}\varepsilon_{pt} \\ \varepsilon_{\beta t}\end{matrix}\middle|\varepsilon_{mt}\right) \sim NID\left[\begin{pmatrix}\mu_{1t} \\ \mu_{2t}\end{pmatrix}, \Sigma_{12|3}\right],$$

$$\Sigma_{12|3} = \begin{bmatrix}\sigma_{pp} & \sigma_{p\beta} \\ \sigma_{p\beta} & \sigma_{\beta\beta}\end{bmatrix} - \begin{bmatrix}\sigma_{pm} \\ \sigma_{\beta m}\end{bmatrix}\sigma_{mm}^{-1}\begin{bmatrix}\sigma_{pm} & \sigma_{\beta m}\end{bmatrix},$$

$$\begin{pmatrix}\mu_{1t} \\ \mu_{2t}\end{pmatrix} = \begin{bmatrix}\sigma_{pm} \\ \sigma_{\beta m}\end{bmatrix}\sigma_{mm}^{-1}\varepsilon_{mt}$$

Conditioning on ε_{mt} will therefore affect equations (3) and (4) as follows:

(B.2)

$$r_{pt} = \alpha_p + \mu_{1t} + \beta_t r_{mt} + \varepsilon_{pt}^{*},$$

$$\beta_t = c + \mu_{2t} + \phi\beta_{t-1} + \boldsymbol{\gamma}'\mathbf{z}_{t-1} + \varepsilon_{\beta t}^{*},$$

$$\begin{bmatrix}\varepsilon_{pt}^{*} \\ \varepsilon_{\beta t}^{*}\end{bmatrix} \sim NID\left[\mathbf{0}, \boldsymbol{\Sigma}_{12|3}\right].$$

It is possible to run the Kalman filter and the Carter and Kohn (1994) simulation smoother on the state space (B.2) and obtain a draw from the conditional posterior distribution of the betas.

B.2 Posterior simulation of α_p

Conditioned on all other parameters ($\overline{\boldsymbol{\theta}}_\alpha$), on the series $\boldsymbol{\beta}_T$ and the data amounts to conditioning on shocks $\varepsilon_{\beta t}$ and ε_{mt} . The relevant conditional distribution of ε_{pt} is:

(B.4)
$$\left(\varepsilon_{pt} \left| \begin{bmatrix} \eta_{pt} \\ u_{mt} \end{bmatrix} \right.\right) \sim NID\left[(\mu_{1t}), \sigma_\varepsilon^{*2}\right]$$
$$\sigma_\varepsilon^{*2} = \sigma_\eta^2 - \begin{bmatrix} \sigma_{\varepsilon\eta} & \sigma_{\varepsilon u} \end{bmatrix} \begin{bmatrix} \sigma_{\eta\eta} & \sigma_{\eta u} \\ \sigma_{\eta u} & \sigma_{uu} \end{bmatrix}^{-1} \begin{bmatrix} \sigma_{\varepsilon\eta} \\ \sigma_{\varepsilon_u} \end{bmatrix}$$
$$\mu_{1t} = \begin{bmatrix} \sigma_{\varepsilon\eta} & \sigma_{\varepsilon u} \end{bmatrix} \begin{bmatrix} \sigma_{\eta\eta} & \sigma_{\eta u} \\ \sigma_{\eta u} & \sigma_{uu} \end{bmatrix}^{-1} \left| \begin{bmatrix} \eta_{pt} \\ u_{mt} \end{bmatrix} \right|.$$

This of course will change the intercept in the first equation:

(B.5)
$$y_{1t} = r_{mt} - \mu_{1t} - \beta_t r_{mt} = \alpha_p + \varepsilon_{pt}^*,$$
$$\varepsilon_{pt}^* \sim N\left(0, \sigma_{1|2,3}^2\right).$$

This result, together with a Gaussian prior pdf for α_p, with moments $\underline{\mu}_\alpha, \underline{\sigma}_\alpha^2$ produces a conditional posterior which is Gaussian:

(B.6)
$$p\left(\alpha_p \mid \overline{\boldsymbol{\theta}}_\alpha, \boldsymbol{\beta}_t \mathbf{r}_{mt}, \mathbf{r}_{pt}\right) = N\left(\overline{\mu}_a, \overline{\sigma}_\alpha^2\right)$$
$$\overline{\mu}_a = \frac{\sum_{t=1}^{T} y_{1t}}{\sigma_\varepsilon^{*2}} + \frac{\underline{\mu}_\alpha}{\underline{\sigma}_\alpha^2}, \overline{\sigma}_\alpha^{2} = \left(\frac{1}{\underline{\sigma}_\alpha^2} + \frac{T}{\sigma_\varepsilon^{*2}}\right)^{-1}.$$

B.3 Posterior simulation of the parameters in the beta equation

The parameters to be drawn are $\boldsymbol{\theta}_\beta = [c, \phi, \Gamma']$. Conditioning on the series of the β_t s and all the other parameters of the model ($\overline{\boldsymbol{\theta}}_\beta$), we then observe the whole sequence of ε_{pt} and u_{mt}. Therefore, the conditional distribution of η_{pt} becomes

(B.7)
$$\left(\eta_{pt}\left|\begin{bmatrix}\varepsilon_{pt}\\ u_{mt}\end{bmatrix}\right.\right) \sim NID\left[(\mu_{2t}), \sigma^{*2}_{\eta|\varepsilon u}\right]$$
$$\sigma^{*2}_{\eta|\varepsilon u} = \sigma^2_\eta - \begin{bmatrix}\sigma_{\eta\varepsilon} & \sigma_{\eta u}\end{bmatrix}\begin{bmatrix}\sigma^2_\varepsilon & \sigma_{\varepsilon u}\\ \sigma_{\varepsilon u} & \sigma^2_u\end{bmatrix}^{-1}\begin{bmatrix}\sigma_{\eta\varepsilon}\\ \sigma_{\eta u}\end{bmatrix}$$
$$\mu_{2t} = \begin{bmatrix}\sigma_{\eta\varepsilon} & \sigma_{\eta u}\end{bmatrix}\begin{bmatrix}\sigma^2_\varepsilon & \sigma_{\varepsilon u}\\ \sigma_{\varepsilon u} & \sigma^2_u\end{bmatrix}^{-1}\begin{bmatrix}\varepsilon_{pt}\\ u_{mt}\end{bmatrix}.$$

This induces an extra intercept term in equation (4) which is accounted for by defining as dependent variable and regressors in this equation:

(B.8)
$$\mathbf{y}_2 = \mathbf{X}_\beta \boldsymbol{\theta}_\beta + \boldsymbol{\eta}^*,$$
$$y_{2t} = \beta_t - \mu_{1t}, \mathbf{x}_{t,\beta} = [1, \beta_{t-1}, \boldsymbol{\Gamma}']'.$$

Therefore, using for $\boldsymbol{\theta}_\beta$ a Gaussian prior distribution with moments $\underline{\boldsymbol{\mu}}_{\theta_\beta}$ and $\underline{\boldsymbol{\Sigma}}_{\theta_\beta}$, we can apply the usual conditional conjugate results and obtain a conditional posterior distribution for $\boldsymbol{\theta}_\beta$ which is Gaussian:

(B.9)
$$p\left(\boldsymbol{\theta}_\beta \mid \overline{\boldsymbol{\theta}}_{\theta_\beta}, \boldsymbol{\beta}_t, \mathbf{r}_{mt}, \mathbf{r}_{pt}\right) = N(\overline{\boldsymbol{\mu}}_{\theta\beta}, \overline{\boldsymbol{\Sigma}}_{\theta_\beta})$$
$$\overline{\boldsymbol{\mu}}_{\theta\beta} = \frac{\mathbf{X}'_\beta \mathbf{y}_1}{\sigma^{*2}_{\eta|}} + \underline{\boldsymbol{\Sigma}}^{-1}_{\theta_\beta}\underline{\boldsymbol{\mu}}_{\theta_\beta}, \overline{\boldsymbol{\Sigma}}_{\theta_\beta} = \left(\frac{\mathbf{X}'_\beta \mathbf{X}_\beta}{\sigma^{*2}_{\eta|}} + \underline{\boldsymbol{\Sigma}}^{-1}_{\theta_\beta}\right)^{-1}.$$

B.4 Posterior simulation of the $\boldsymbol{\Lambda}$ parameters

Conditioning on the β_t sequence and all the other parameters of the model ($\overline{\boldsymbol{\theta}}_\Lambda$), then it is as if we observe ε_{pt} and η_{pt}. Therefore the conditional distribution of u_{mt} is:

(B.10)
$$\left(u_{mt} \middle| \begin{bmatrix} \varepsilon_{pt} \\ \eta_{pt} \end{bmatrix} \right) \sim NID\left[(\mu_{3t}), \sigma_u^{*2} \right]$$
$$\sigma_u^{*2} = \sigma_u^2 - \begin{bmatrix} \sigma_{u\varepsilon} & \sigma_{u\eta} \end{bmatrix} \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} \\ \sigma_{\varepsilon\eta} & \sigma_\eta^2 \end{bmatrix}^{-1} \begin{bmatrix} \sigma_{u\varepsilon} \\ \sigma_{u\eta} \end{bmatrix}$$
$$\mu_{3t} = \begin{bmatrix} \sigma_{u\varepsilon} & \sigma_{u\eta} \end{bmatrix} \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} \\ \sigma_{\varepsilon\eta} & \sigma_\eta^2 \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_{pt} \\ \eta_{pt} \end{bmatrix}.$$

this induces an extra intercept term in equation (5) which will be then accounted for by defining the dependent variable and regressors of this equation:

(B.11)
$$\mathbf{y}_3 = \mathbf{Z}_{-1}\boldsymbol{\Lambda} + \mathbf{u}^*,$$
$$y_{3t} = r_{mt} - \mu_{3t}.$$

Therefore, using for $\boldsymbol{\Lambda}$ a Gaussian prior with moments $\underline{\boldsymbol{\mu}}_\Lambda$ and $\underline{\boldsymbol{\Sigma}}_\Lambda$, we can apply the usual conditional conjugate results and obtain a conditional posterior distribution for $\boldsymbol{\Lambda}$ which is Gaussian:

(B.12)
$$p\left(\boldsymbol{\Lambda} \mid \overline{\boldsymbol{\theta}}_\Lambda, \boldsymbol{\beta}_T, \mathbf{r}_{mt}, \mathbf{r}_{pt}\right) = N\left(\overline{\boldsymbol{\mu}}_\Lambda, \overline{\boldsymbol{\Sigma}}_\Lambda\right)$$
$$\overline{\boldsymbol{\mu}}_\Lambda = \frac{\mathbf{Z}_{-1}^{'}\mathbf{y}_3}{\sigma_\upsilon^{*2}} + \underline{\boldsymbol{\Sigma}}_\Lambda^{-1}\underline{\boldsymbol{\mu}}_\Lambda, \overline{\boldsymbol{\Sigma}}_\Lambda = \left(\frac{\mathbf{Z}_{-1}^{'}\mathbf{Z}_{-1}}{\sigma_u^{*2}} + \underline{\boldsymbol{\Sigma}}_\Lambda^{-1} \right)^{-1}.$$

B.5 Posterior simulation of the $\boldsymbol{\Sigma}$ parameters

Conditional on the data, the betas and the remaining parameters ($\bar{\boldsymbol{\theta}}_{\Sigma}$), using a Wishart prior for $\boldsymbol{\Sigma}^{-1}$, we obtain, via the usual conjugate results, a Wishart conditional posterior:

$$p\left(\boldsymbol{\Sigma}^{-1} \mid \bar{\boldsymbol{\theta}}_{\Sigma}, \boldsymbol{\beta}_T, \mathbf{r}_{mt}, \mathbf{r}_{pt}\right) = W_n\left(\mathbf{H} \mid \bar{\nu}, \bar{\mathbf{S}}\right),$$

(B.13) $$\bar{\nu} = T + \underline{\nu}, \bar{\mathbf{S}} = \mathbf{S} + \underline{\mathbf{S}},$$

$$\mathbf{S} = \sum_{t=1}^{T} \boldsymbol{\eta}_t \boldsymbol{\eta}_t'.$$