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Ryan S. Mattson* Philippe de Peretti‡

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Abstract

In this paper, we use semi-nonparametric tests for weak separability introduced by Barnett and de Peretti (2009) to check for natural clustering of monetary goods. Using data from the Center of Financial Stability (CFS) over January 1967 to December 2012, we test for the separability of 6 different monetary aggregates, both narrow (e.g. DM1 or DM2M) and broad (e.g. DM4- or DM4). Regarding standard nonparametric tests (Varian 1982, 1983), the approach used has several advantages: i) It is stochastic, dealing with the significance of the violations of GARP, ii) It is based on a necessary and sufficient condition for weak separability. On five different samples, our results suggest that separability is found only for very liquid assets, i.e. the assets included in the DM1 aggregate. For all samples, the separability of broad monetary aggregates is strongly rejected. Moreover the utility maximization tests detect high instability in monetary behaviors, during June 1974 to November 1982, as well as during the sub-prime crisis.

*Contact Author. Department of Economics, Rhodes College, 320 Buckman Hall 2000 N Parkway, Memphis, TN. tel. 901-843-3122, mail mattsonr@rhodes.edu

†Centre d'Economie de la Sorbonne (CES), Université Paris1 Panthéon-Sorbonne, 106-112 Boulevard de l'Hôpital, 75013 Paris, France. Tel. 00 33 1 44 07 87 46, mail: philippe.de-peretti@univ-paris1.fr

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1 Introduction

Tests of weak separability have received a great deal of attention in these recent years. Indeed, they are of key importance, since weak separability is the corner stone of modern aggregation theory. Weak separability is a necessary condition for existence of an aggregate, ensuring that all substitution effects are internalized (Barnett, 1980; Deaton and Muellbauer, 1980; Blackorby et al. 1998). It also returns important information about the structure of preferences, more specifically about the natural clustering over goods and assets. The latter aspect is widely used in monetary economics to define money[1]. Within this framework, defining money amounts to finding the monetary assets rationalized by a weakly separable utility function. For instance, if one finds a group of assets containing notes and coins and less liquid ones as savings deposits rationalized by a separable monetary sub-utility function, then the group can be defined as 'money'. In other words, to define money, one must find the assets valued by microeconomic agents for their role as money. Empirical applications can be found in Swofford and Whitney (1987, 1988), Drake (1997) or Fisher and Fleissig (1997) among others. Generally, results support the separability of, at least, a narrow aggregate of money.

To check for weak separability, most studies have focused on nonparametric tests, thus avoiding the Denny and Fuss (1977) critique. Early studies, like Swofford and Whitney (1987, 1988) use tests developed by Varian (1982, 1983) which employ Afriat (1967, 1973) results within the Samuelson's revealed preference framework. The Varian test for separability is sequential in that it tests different groups of prices and quantities for utility maximization using the well-known Generalized Axiom of Revealed Preference (GARP). Since GARP is nonstochastic and the Varian test is based on a only sufficient condition, and since it is strongly biased toward rejection (Barnett and Choi, 1989) several researchers have extended the analysis. A recent sample of research[2] includes Swofford and Whitney (1994), Fleissig and Whitney (2003, 2005), Elger and Jones (2008), Cherchye et al (2012), and Barnett and de Peretti (2009), based upon de Peretti (2007).

The goal of this paper is to provide researchers with sound econometric results in order to build theoretically consistent monetary aggregates. For this, we implement the Barnett and de Peretti (2009) test to check for the separability of 6 different monetary aggregates, narrow and broad, as defined by the Center for Financial Stability[3] (CFS) over the period January 1967

[1] For justifications of money in the utility function, see Feenstra (1986), or Poterba and Rotemberg (1987).

[2] See also Varian (1985, 1990).

[3] http://www.centerforfinancialstability.org/

to December 2012. The Barnett and de Peretti (2009) procedure is a necessary and sufficient semi-nonparametric test, partly based on GARP. It uses results of the two-step procedure described in de Peretti (2005, 2007). First, GARP is used to test for compliance with both the utility maximization and the sub-utility maximization programs. If violations appear, under an additive measurement error framework, a quadratic routine is applied that computes the minimal theoretical adjustment in the quantity data so that the two maximization programs hold. The theoretical adjustment is then tested for its significance by comparing it to an estimate of the 'true' extremal measurement errors in the data. At this step, compared to Barnett and de Peretti (2009), we alter the quadratic routine, changing the transitive constraints, thus allowing for more flexibility and returning smaller adjustments. If the data pass step-one, then separability is tested using a necessary and sufficient condition which exploits the independence between the marginal rate of substitution between two assets of the cluster and assets/goods outside the cluster using a multivariate independence test.

The main results of this paper are as follows: i) For three of the five samples used in this study, there exist monetary sub-utility functions, even corresponding to the DM4- of DM4 aggregate, but only the sub-utility functions over DM1 yield evidence of weak separability. No other function is supported by this test, ii) Tests detect a high instability in monetary behaviors over the period June 1974 to November 1982 and September 1991 to December 2012. On this latter period we hypothesize that either the sub-prime crisis caused ruptures in the monetary utility function, or the 'easy money' period preceding the crisis led agents to lose rationality over money.

The paper is structured as follows. In Section 2 we introduce the classical nonparametric tests for utility maximization and present the Barnett and de Peretti (2009) test. Modifications of the set of constraints is introduced, as well as the multivariate independence test. In Section 3, we present the data and implement the tests. At last, in Section 4 we conclude and discuss our results and extensions.

2 Nonparametric tests of utility maximization and weak separability

We first introduce nonparametric tests of utility maximization and weak separability, as defined by Varian (1982, 1983). Subsequently, the Barnett and de Peretti (2009) procedure is presented that deals with two major shortcomings of the Varian approach: i) The tests are non-stochastic, and ii) the tests are based on an only sufficient condition.

2.1 Nonparametric tests of weak separability

Let $\mathbf{X}$ be a $(T \times k)$ matrix of real quantities, and let $\mathbf{x}_i = (x_{i1}, x_{i2}, ..., x_{ik})'$ be the ith row of the matrix, $i \in (1, T)$. Define $\mathbf{P}$ as a $(T \times k)$ matrix of corresponding prices, where similarly $\mathbf{p}_i = (p_{i1}, p_{i2}, ..., p_{ik})'$ is the ith row of the matrix, $i \in (1, T)$. Let there be two partitions of $\mathbf{X}$, the $(T \times a)$ $\mathbf{X}^{(1)}$ matrix, $a \in (1, k-1)$, with $\mathbf{x}_i^{(1)} = (x_{i1}, x_{i2}, ..., x_{ia})'$, and the $(T \times (k-a))$ $\mathbf{X}^{(2)}$ matrix with $\mathbf{x}_i^{(2)} = (x_{i(a+1)}, x_{i(a+2)}, ..., x_{ik})'$, $\mathbf{X}^{(1)} \cup \mathbf{X}^{(2)} = \mathbf{X}$ and $\mathbf{X}^{(1)} \cap \mathbf{X}^{(2)} = \varnothing$. $\mathbf{P}^{(1)}$ and $\mathbf{P}^{(2)}$ are the corresponding prices with $\mathbf{p}_i^{(1)} = (p_{i1}, p_{i2}, ..., p_{ia})'$ and $\mathbf{p}_i^{(2)} = (p_{i(a+1)}, p_{i(a+2)}, ..., p_{ik})'$. Now, define weak separability over $\mathbf{X}^{(1)}$, as follows:

Definition 1. *There is weak separability if i) There exists a utility function (1) rationalizing the data, and ii) This latter admits a rewriting (2).*

$$U_i = U(\mathbf{x}_i), \ i = 1, 2, ..., T \tag{1}$$

$$U_i = V\left(\mathbf{x}_i^{(2)}, f(\mathbf{x}_i^{(1)})\right), \ i = 1, 2, ..., T \tag{2}$$

where:
$U()$ *is the overall utility function,*
$V()$ *is a strictly increasing macro-function,*
$f()$ *is the sub-utility function (micro-function).*

Clearly, the weak separability of preferences is a necessary and sufficient condition for an aggregate to exist (Deaton and Muellbauer, 1980). Indeed, if weak separability is found, then the marginal rate of substitution between any two goods of $\mathbf{X}^{(1)}$ is independent of the goods outside the cluster, ensuring that all substitutions effects are internalized.

$$\partial \left(\frac{\frac{\partial U(\mathbf{x}_i)}{\partial x_{ij}}}{\frac{\partial U(\mathbf{x}_i)}{\partial x_{il}}} \right) / \partial x_{im} = 0 \tag{3}$$

for $j, l \in (1, a)$, $j \neq l$; $m = a+1, ..., k$; $i = 1, ..., T$.
Weak separability also returns key information about natural clustering over goods. For instance, applied to the monetary framework, defining money, i.e. selecting the assets entering a monetary aggregate, amounts to finding very liquid ones as coins and notes and less liquid ones such as savings accounts and time deposits, rationalized by a weakly separable (monetary) sub-utility

function. For weak separability to exist, three conditions must be fulfilled: $U()$, $f()$ and $V()$ must exist, and testing for weak separability therefore reduces to a three-step test of utility maximization. Varian (1982, 1983) introduced such a procedure, basing each step on the Generalized Axiom of Revealed Preference (GARP) introduced hereafter. Define the three following binary relations as follows, $\mathbf{x}_i$ is said to be strictly directly revealed preferred to $\mathbf{x}_j$ if $\mathbf{p}_i \cdot \mathbf{x}_i > \mathbf{p}_i \cdot \mathbf{x}_j$, written $\mathbf{x}_i P^0 \mathbf{x}_j$; $\mathbf{x}_i$ is said to be directly revealed preferred to $\mathbf{x}_j$ if $\mathbf{p}_i \cdot \mathbf{x}_i \geq \mathbf{p}_i \cdot \mathbf{x}_j$, written $\mathbf{x}_i R^0 \mathbf{x}_j$; $\mathbf{x}_i$ is said to be revealed preferred to $\mathbf{x}_j$ if $\mathbf{x}_i R^0 \mathbf{x}_m$,$\mathbf{x}_m R^0 \mathbf{x}_k, ..., \mathbf{x}_p R^0 \mathbf{x}_j$, written $\mathbf{x}_i R \mathbf{x}_j$, where R is the transitive closure of R^0. GARP is then defined as follows:

Definition 1 (GARP). *For a couple of observations* $(i,j) \in (1,T), i \neq j : \mathbf{x}_i R \mathbf{x}_j \Rightarrow \mathbf{p}_j \cdot \mathbf{x}_j \leq \mathbf{p}_j \cdot \mathbf{x}_i$, *or* $\mathbf{x}_i R \mathbf{x}_j$ *implies **not*** $\mathbf{p}_j \cdot \mathbf{x}_j > \mathbf{p}_j \cdot \mathbf{x}_i$.

GARP states that if $\mathbf{x}_i$ is revealed preferred to $\mathbf{x}_j$, and if at prices $\mathbf{p}_j$, $\mathbf{x}_i$ is still affordable, then it still must be chosen. If instead, $\mathbf{x}_j$ is chosen it is a GARP violation. Using GARP, Varian (1982) proved that:

Theorem 1 (Varian 1982). *For a set* $(\mathbf{x}_i, \mathbf{p}_i)_{i=1}^T$, *the three following conditions are equivalent:*

i) *There exists a locally non-satiated utility function* $U(.)$ *that rationalizes the data,*

ii) *There exist strictly positive utility indices* U_i *and marginal income indices* λ_i *that satisfy* $\forall (i,j) \in (1,T)$ *the Afriat inequalities (4),*

$$U_i \leqslant U_j + \lambda_j(\mathbf{p}_j \cdot \mathbf{x}_i - \mathbf{p}_j \cdot \mathbf{x}_j) \tag{4}$$

iii) *The data satisfy GARP*, i.e. $\forall (i,j) \in (1,T), i \neq j : \mathbf{x}_i R \mathbf{x}_j$ *implies **not*** $\mathbf{p}_j \cdot \mathbf{x}_j > \mathbf{p}_j \cdot \mathbf{x}_i$.

Based on Theorem 1, testing for the weak separability of $\mathbf{X}^{(1)}$ is a straightforward check of whether the following three conditions hold:

Condition 1. *GARP holds for* $(\mathbf{x}_i, \mathbf{p}_i)_{i=1}^T$, *that is* $U(.)$ *exists,*

Condition 2. *GARP holds for* $(\mathbf{x}_i^{(1)}, \mathbf{p}_i^{(1)})_{i=1}^T$, *that is* $f(.)$ *exists,*

Condition 3. *GARP holds for* $((\mathbf{x}_i^{(2)}, U_i), (\mathbf{p}_i^{(2)}, \lambda_i^{-1}))_{i=1}^T$, *where* U_i *and* λ_i *are strictly positive indices satisfying (4) for* $(\mathbf{x}_i^{(1)}, \mathbf{p}_i^{(1)})_{i=1}^T$, *that is* $\mathbf{X}^{(1)}$ *is weakly separable in* $U(.)$.

Barnett and de Peretti (2009) discuss two extensions of the above approach in order to: i) Take into account the non stochasticity of GARP, ii) Propose an alternate Condition 3, replacing the only sufficient condition one[4].

2.2 Necessary and sufficient tests for weak separability

Dealing with the non stochasticity of GARP, we follow Barnett and de Peretti (2009) and de Peretti (2005, 2007). Assume that under the null, $\mathbf{X}^*$ and $\mathbf{X}^{*(1)}$ are respectively generated by a utility and a sub-utility function, but are unobservable. Only $\mathbf{X}$ is observed, being related to $\mathbf{X}^*$ by an additive relation:

$$x_{ij} = x_{ij}^* + \varepsilon_{ij} \tag{5}$$

Where:

ε_{ij} is iid with zero mean and variance $\sigma^2_{\varepsilon_j}$, with distribution function $F_j(x)$. The distribution $F_j(x)$ is max and min-stable, $j = 1, ..., k$.

To jointly test for Conditions 1 and 2, they suggest a procedure that consists of:

i) Computing the minimal adjustment in the quantity data to produce a data set compliant with both the overall utility maximization and the sub-utility maximization programs, referred to as the adjustment procedure,

ii) Testing the significance of that adjustment, referred to as the adjustment procedure,

Thus, if violations appear when testing for Conditions

The program is an extension of de Peretti (2005) for the overall maximization program. If violations appear when testing for $U(.)$, rebuild a data set $(\mathbf{z}_i, \mathbf{p}_i)_{i=1}^T$ compliant with utility maximization such that the distance with the observed one $(\mathbf{x}_i, \mathbf{p}_i)_{i=1}^T$ is minimal. This is achieved by minimizing over z_{ij} the following quadratic form under a set of transitive constraints[5]:

$$\begin{aligned} obj &= \min_{z_{ij}} \sum_{i=1}^{T} \sum_{j=1}^{k} (x_{ij} - z_{ij})^2 \qquad (6) \\ \text{subject to} &: \forall (i,j) \in (1,T) : \mathbf{z}_i R \mathbf{z}_j \text{ implies not } \mathbf{z}_j P^0 \mathbf{z}_i \end{aligned}$$

To solve (6), a stepwise procedure is developed that uses all the information content in the transitive closure matrix R.

[4]See Varian (1983) and Fleissig and Whitney (2003).

[5]See also Varian (1985).

Definition 6. *Two observations* $\mathbf{x}_i$ *and* $\mathbf{x}_j$ *satisfy the binary relation* $\mathbf{x}_i VR \mathbf{x}_j$ *if there exists a sequence between* $\mathbf{x}_i$ *and* $\mathbf{x}_j$ *such that* $\mathbf{x}_i R \mathbf{x}_k$ *and* $\mathbf{x}_k P^0 \mathbf{x}_i$, $\mathbf{x}_k R \mathbf{x}_m$ *and* $\mathbf{x}_m P^0 \mathbf{x}_k, ..., \mathbf{x}_n R \mathbf{x}_j$ *and* $\mathbf{x}_j P^0 \mathbf{x}_n$. *Such a sequence is called a violation chain.*

Definition 7. *Two observations* $\mathbf{x}_i$ *and* $\mathbf{x}_j$ *satisfy the binary relation* $\mathbf{x}_i SR \mathbf{x}_j$ *if* $S(i) = S(j)$, *where* $S(i) = \left(\sum_{j=1}^{T} r_{ij}\right) - 1$ *is a function returning the sum* m *of the* i*th row and the transitive closure matrix* R, m *returning how many bundles* $\mathbf{x}_i$ *is revealed preferred to.*

Interestingly, $\mathbf{x}_i VR \mathbf{x}_j$ implies $\mathbf{x}_i SR \mathbf{x}_j$. The major implication is that bundles violating GARP can be classified in n independent sets $B_l, l = 1, 2..., n$, each set corresponding to a particular rupture in the preference chain, i.e. each set corresponding to a different m. Using this result, the stepwise procedure can therefore be used:

Step 1. Test $D = (\mathbf{x}_i, \mathbf{p}_i)_{i=1}^{T}$ for compliance with GARP. If violations appear go to Step 2, otherwise, stop the loop.

Step 2. Search for clusters of violations, by classifying all bundles violating GARP in n independent sets B_l. In each set, the bundles violating GARP are related by $\mathbf{x}_i VR \mathbf{x}_j$ and $\mathbf{x}_i SR \mathbf{x}_j$. Search for the one, defined as B_1 such that all bundles included in, are all candidates to be at the same higher place in the preference chain. Go to Step 3

Step 3. For all bundles $\mathbf{x}_i \in B_1$, solve the quadratic program and store the objective function. Go to Step 4.

$$obj_i = \min_{z_{ij}} \sum_{j=1}^{k} (x_{ij} - z_{ij})^2 \tag{7}$$

subject to :

$$\left| \begin{array}{l} \mathbf{p}_i \cdot \mathbf{x}_j \geq \mathbf{p}_i \cdot \mathbf{z}_i, \forall \mathbf{x}_j \text{ such that} \\ \quad \mathbf{x}_j R \mathbf{x}_i \text{ implies not } \mathbf{x}_i P^0 \mathbf{x}_j \end{array} \right. \tag{C.1}$$

$$\left| \ \mathbf{p}_m \cdot \mathbf{x}_m \leq \mathbf{p}_m \cdot \mathbf{z}_i, \forall \mathbf{x}_m \in B_1,\ m \neq i \right. \tag{C.2}$$

Step 4. Select the bundle having the minimal objective function, and let $\widehat{\mathbf{z}}_i$ be the bundle solution.

Step 5. In D, replace $\mathbf{x}_i$ by $\widehat{\mathbf{z}}_i$. Go to Step 1.

The five above steps are referred to as the 'core' algorithm, which is designed to sequentially remove all the violations of GARP. Note that different sets of constraints are used compared with de Peretti (2005, 2007) and Barnett and de Peretti (2009). The C.2 is equivalent, but simplified, but the C.1 is quite different. Indeed, in the original program, the overall expenditure in i was forced to remain unchanged ($\mathbf{p}_i \cdot \mathbf{z}_i = \mathbf{p}_i \cdot \mathbf{x}_i$) in order for $\mathbf{z}_i$ to i) Remain at a given place in the reference chain, ii) Not cause new violations with bundles located above in the preference chain. Here, we use exactly the same idea but allow for more flexibility, allowing the overall budget to change, replacing $\mathbf{p}_i \cdot \mathbf{z}_i = \mathbf{p}_i \cdot \mathbf{x}_i$ by $\mathbf{p}_i \cdot \mathbf{x}_j \geq \mathbf{p}_i \cdot \mathbf{z}_i, \forall \mathbf{x}_j$ such that $\mathbf{x}_j R \mathbf{x}_i$ implies not $\mathbf{x}_i P^0 \mathbf{x}_j$.

Barnett and de Peretti (2009) extend the above routine to produce data compliant with both the utility and sub-utility maximization. In their routine, they:

i) Adjust data to be compliant with the sub-utility,

ii) Adjust data to be compliant with the overall utility under the additional constraint that data remain compliant with the sub-utility.

This is achieved by solving over z_{ij} the following program:

$$obj = \min_{z_{ij}} \sum_{i=1}^{T} \sum_{j=1}^{k} \left(x_{ij} - z_{ij}\right)^2 \tag{8}$$

Subject to :

$$\left| \; \forall (i,j) \in (1,T) : \mathbf{z}_i R \mathbf{z}_j \Rightarrow \mathbf{p}_j \cdot \mathbf{z}_j \leq \mathbf{p}_j \cdot \mathbf{z}_i \right. \tag{C.1}$$

$$\left| \begin{array}{l} \forall (i,j) \in (1,T) : \mathbf{z}_i^{(1)} R \mathbf{z}_j^{(1)} \Rightarrow \mathbf{p}_j^{(1)} \cdot \mathbf{z}_j^{(1)} \leq \mathbf{p}_j^{(1)} \cdot \mathbf{z}_i^{(1)} \\ \text{where:} \\ \mathbf{z}_i^{(1)} = (z_{i1}, z_{i2}, ..., z_{ia})' \text{ and } \mathbf{p}_i^{(1)} = (p_{i1}, p_{i2}, ..., p_{ia})' \end{array} \right. \tag{C.2}$$

This uses the core algorithm in the following way:

i) If violations of GARP are found for the observed $(\mathbf{x}_i^{(1)}, \mathbf{p}_i^{(1)})_{i=1}^T$, the core algorithm is run, replacing $D = (\mathbf{x}_i, \mathbf{p}_i)_{i=1}^T$ by $D^{(1)} = (\mathbf{x}_i^{(1)}, \mathbf{p}_i^{(1)})_{i=1}^T$ and (7) by (9)

$$obj_i = \min_{z_{ij}^{(1)}} \sum_{j=1}^{a} \left(x_{ij}^{(1)} - z_{ij}^{(1)}\right)^2 \tag{9}$$

Subject to :

$$\left| \begin{array}{c} \mathbf{p}_i^{(1)} \cdot \mathbf{x}_j^{(1)} \geq \mathbf{p}_i^{(1)} \cdot \mathbf{z}_i^{(1)}, \forall \mathbf{x}_j^{(1)} \\ \text{such that } \mathbf{x}_j^{(1)} R \mathbf{x}_i^{(1)} \text{ implies not } \mathbf{x}_i^{(1)} P^0 \mathbf{x}_j^{(1)} \end{array} \right. \tag{C.1}$$

$$\left| \; \mathbf{p}_m^{(1)} \cdot \mathbf{x}_m^{(1)} \leq \mathbf{p}_m^{(1)} \cdot \mathbf{z}_i^{(1)}, \forall \mathbf{x}_m^{(1)} \in B_1, \; m \neq i \right. \tag{C.2}$$

The core algorithm will return a set $(\widehat{\mathbf{z}}_i^{(1)}, \mathbf{p}_i^{(1)})_{i=1}^T$, compliant with the sub-utility maximization. For simplicity, rename $\widehat{\mathbf{z}}_i^{(1)}$ as $\boldsymbol{\xi}_i$ and then $D^2 = \left((\boldsymbol{\xi}_i, \mathbf{x}_i^{(2)}), \; (\mathbf{p}_i^{(1)}, \mathbf{p}_i^{(2)})\right)_{i=1}^T$.

ii) If violations appear when testing GARP on D^2, compute a set compliant with both the sub-utility and the overall utility by using the core algorithm and replacing D by D^2 and (7) by (10).

$$obj_i \quad = \quad \min_{z_{ij}^{(1)}, z_{ij}^{(2)}} \left[\sum_{j=1}^{a} \left(x_{ij} - z_{ij}^{(1)} \right)^2 + \sum_{j=a+1}^{k} \left(x_{ij} - z_{i(j-a)}^{(2)} \right)^2 \right] \tag{10}$$

Subject to :

$$\left| \begin{array}{l} \mathbf{p}_i^{(1)} \cdot \boldsymbol{\xi}_j + \mathbf{p}_i^{(2)} \cdot \mathbf{x}_j^{(2)} \geq \mathbf{p}_i^{(1)} \cdot \mathbf{z}_i^{(1)} + \mathbf{p}_i^{(2)} \cdot \mathbf{z}_i^{(2)}, \\ \qquad \forall (\boldsymbol{\xi}_j, \mathbf{x}_j^{(2)}) \text{ such that} \\ (\boldsymbol{\xi}_j, \mathbf{x}_j^{(2)}) R (\boldsymbol{\xi}_i, \mathbf{x}_i^{(2)}) \text{ implies not } (\boldsymbol{\xi}_i, \mathbf{x}_i^{(2)}) P^0 (\boldsymbol{\xi}_j, \mathbf{x}_j^{(2)}) \end{array} \right. \tag{C.1}$$

$$\left| \begin{array}{l} \mathbf{p}_m^{(1)} \cdot \boldsymbol{\xi}_m + \mathbf{p}_m^{(2)} \cdot \mathbf{x}_m^{(2)} \leq \mathbf{p}_m^{(1)} \cdot \mathbf{z}_i^{(1)} + \mathbf{p}_m^{(2)} \cdot \mathbf{z}_i^{(2)}, \\ \qquad \forall (\mathbf{p}_m^{(1)} \cdot \boldsymbol{\xi}_m) \in B_1, \ m \neq i \end{array} \right. \tag{C.2}$$

$$\left| \begin{array}{l} \mathbf{p}_i^{(1)} \cdot \mathbf{x}_j^{(1)} \geq \mathbf{p}_i^{(1)} \cdot \mathbf{z}_i^{(1)}, \forall \mathbf{x}_j^{(1)}, \text{ such that} \\ \quad \mathbf{x}_j^{(1)} R \mathbf{x}_i^{(1)} \text{ implies not } \mathbf{x}_i^{(1)} P^0 \mathbf{x}_j^{(1)} \end{array} \right. \tag{C.3}$$

$$\left| \ \mathbf{p}_m^{(1)} \cdot \mathbf{x}_m^{(1)} \leq \mathbf{p}_m^{(1)} \cdot \mathbf{z}_i^{(1)}, \forall \mathbf{x}_m^{(1)} \in B_1, \ m \neq i \right. \tag{C.4}$$

The above program returns a set $\left((\widehat{\mathbf{z}}_i^{(1)}, \widehat{\mathbf{z}}_i^{(2)}), (\mathbf{p}_i^{(1)}, \mathbf{p}_i^{(2)}) \right)_{i=1}^{T}$, compliant with both the utility function and the sub-utility maximization programs. Therefore, $\widehat{\mathbf{A}} = (\mathbf{X} - \widehat{\mathbf{Z}})$ is the minimal adjustment in the data so that Conditions 1 and 2 of the Varian sequence hold. $\widehat{\mathbf{A}}$ is referred to as the minimal 'theoretical' adjustment. To test the significance of this theoretical adjustment, we use a test that tracks excess adjustments in some goods. The particular relevance of the test is that it allows discriminating between significant and non-significant violations (excess adjustment). Following Barnett and de Peretti (2009) we compare the extremal theoretical adjustments to some estimates of the extremal 'true' measurement errors in the data. This can be done in two ways.

Note that (5) can be seen as the measurement equation of a state-space model, whose form is given by the following equations for a single good x_{ij} :

$$x_{ij} \quad = \quad x_{ij}^* + \varepsilon_{ij}, \varepsilon_{ij} \sim iid(0, \sigma_{\varepsilon_j}^2) \tag{11}$$

$$x_{(i+1)j}^* \quad = \quad \mu_{ij} + \nu_{ij} + \xi_{ij}, \xi_{ij} \sim iid(0, \sigma_{\xi_j}^2) \tag{12}$$

$$\nu_{(i+1)j} \quad = \quad \nu_{ij} + \zeta_{ij}, \zeta_{ij} \sim iid(0, \sigma_{\zeta_j}^2) \tag{13}$$

The above model is just the Local Linear Trend model studied by many authors within the structural time series framework (see e.g. Durbin and Koopman 2001). In (11), $\sigma_{\varepsilon_j}^2$ is the variance of the measurement error term, and ε_{ij} the measurement error for good j in period

i, x_{ij}^* is the unobserved quantity (state variable). Let $\widehat{Max}_j$ be the maximal (normalized) adjustment for good $j \in (1, k)$, i.e. $\widehat{Max}_j = \max(\widehat{a}_{1j}\widehat{\sigma}_{\varepsilon_j}^{-1}, \widehat{a}_{2j}\widehat{\sigma}_{\varepsilon_j}^{-1}, ..., \widehat{a}_{Tj}\widehat{\sigma}_{\varepsilon_j}^{-1})$, and $\widehat{Min}_j$ the minimal (normalized) adjustment, i.e. $\widehat{Min}_j = \min(\widehat{a}_{1j}\widehat{\sigma}_{\varepsilon_j}^{-1}, \widehat{a}_{2j}\widehat{\sigma}_{\varepsilon_j}^{-1}, ..., \widehat{a}_{Tj}\widehat{\sigma}_{\varepsilon_j}^{-1})$, where $\widehat{\sigma}_{\varepsilon_j}^{-1}$ is the Kalman filter estimate of $\sigma_{\varepsilon_j}^{-1}$. Following Guégan (2003), under normality assumptions, the two extrema are within the domain of attraction of the Gumbel law:

$$\textit{Gumbel (type I)} : G_1(x) = \exp(-\exp(-x)), \quad x \in \mathbb{R}. \tag{14}$$

Thus, testing for the null of the non-significance of the theoretical adjustments can be easily done by computing the two p-values, for a good j :

$$1 - \exp(-\exp(-y_{1j})) \tag{15}$$

for the maxima, and:

$$1 - \exp(-\exp(y_{2j})) \tag{16}$$

for the minima.

Where $y_{1j} = (\widehat{Max}_j - c_{1j})b_j$ and $y_{2j} = (\widehat{Min}_j - c_{2j})b_j$, where c_{1j} and c_{2j} $(c_{1j} = -c_{2j})$ are the location parameters, and b_j is a scale parameter defined as:

$$c_{1j} = (2\ln T)^{1/2} - \frac{\ln\ln T + \ln 4\pi}{2(2\ln T)^{1/2}} \tag{17}$$

$$b_j = (2\ln T)^{-1/2} \tag{18}$$

Under weaker assumptions, or if the law of the measurement error is unknown, an alternative strategy can be used, based on the quantiles of the distribution of the extremal measurement error computed in (11). The quantiles of the extremal adjustments are computed by using simulations, based on the so-called simulation smoother (de Jong and Shephard, 1995). In the latter, smoothed errors are generated conditional on the observed series $(x_{1j}, x_{2j}, ..., x_{Tj})$, which makes sense regarding the null assumption. Following Durbin and Koopman (2001), define: $T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $M = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} \sigma_{\xi_j}^2 & 0 \\ 0 & \sigma_{\zeta_j}^2 \end{bmatrix}$, then the recursions for the Kalman filter are given by the following filtering equations:

$$\left| \begin{array}{ll} v_i = x_{ij} - M\alpha_{ij} & F_i = Z_i P_i Z_i' + \sigma_{\varepsilon_j}^2 \\ K_i = T_i P_i Z_i' F_i^{-1} & L_i = T_i - K_i Z_i \\ \alpha_{(i+1)j} = T_i\alpha_i + K_i v_i & P_{(i+1)j} = T_i P_i L_i' + Q \end{array} \right. , i = 1, 2, ..., T \tag{19}$$

(19) returns the filtered states $\alpha_{(i+1)j}$.

Now, to generate a sample of smoothed errors $\widetilde{\varepsilon}_{ij}$, the simulations smoothing recursions are given by the following set of equations, solved recursively:

$$\left|\begin{array}{r}\widetilde{r}_{i-1} = M'F_i^{-1}v_i - \widetilde{W}_i C_i^{-1} d_i + L_i'\widetilde{r}_i \\ \widetilde{N}_{i-1} = M_i'F_i^{-1}M_i + \widetilde{W}_i' C_i^{-1}\widetilde{W}_i + L_i'\widetilde{N}_i L_i \\ \widetilde{W}_i = \sigma^2_{\varepsilon_j}(F_i^{-1}Z_i - K_i'\widetilde{N}_i L_i) \\ d_i \sim N(0, C_i) \\ C_i = \sigma^2_{\varepsilon_j} - \sigma^2_{\varepsilon_j}\widetilde{D}_i\sigma^2_{\varepsilon_j} \\ \widetilde{D}_i = F_i^{-1} + K_i'\widetilde{N}_i K_i \\ \widetilde{\varepsilon}_{ij} = d_i + \sigma^2_{\varepsilon_j}(F_i^{-1}v_i - K_i')\widetilde{r}_i\end{array}\right. , \left|\begin{array}{l} i = T, ..., 1 \\ \widetilde{r}_T = 0 \\ \widetilde{N}_T = 0\end{array}\right. \tag{20}$$

Hence to generate series of extrema for goods j, $j = 1, ..., k$, we estimate (11) by maximizing a (diffuse) log-likelihood, solve (19) and then solve (20) a large of number of times, each time storing $\max(\widetilde{\varepsilon}_{1j}, \widetilde{\varepsilon}_{2j}, ..., \widetilde{\varepsilon}_{Tj})$ and $\min(\widetilde{\varepsilon}_{1j}, \widetilde{\varepsilon}_{2j}, ..., \widetilde{\varepsilon}_{Tj})$. We then compute the quantiles at 90%, 95% and 99% of the distribution for the two extrema. To test the significance of the theoretical adjustment, we compare for each j the theoretical extremal adjustments to a given corresponding quantile, and reject the null if the theoretical adjustment exceeds the quantile at a given threshold.

In this paper, we use the test based on the simulation smoother, and if the data pass the above test, then we conclude at standard thresholds that we have both an overall maximization program and a sub-utility maximization program.

To test for weak separability of the sub-utility function, we directly use (3), i.e. we test for the independence between the marginal rate of substitution among any two goods of the group, and the goods outside the group. Since, the marginal rate of substitution is unknown, we use the Konyus and Byushgens Lemma:

Lemma 1 (Konyus and Byushgens). *Suppose $f(.)$ is differentiable and each $\mathbf{x}_i^{(1)}$ is a solution of the maximization program for $f(.)$, i.e. GARP holds for $(\mathbf{x}_i^{(1)}, \mathbf{p}_i^{(1)})_{i=1}^T$, then:*

$$\frac{\mathbf{p}_i^{(1)'}}{\mathbf{p}_i^{(1)} \cdot \mathbf{x}_i^{(1)}} = \frac{\nabla f(\mathbf{x}_i^{(1)})}{\mathbf{x}_i^{(1)} \cdot \nabla f(\mathbf{x}_i^{(1)})}, i = 1, ..., T \tag{21}$$

where: $\nabla f(\mathbf{x}_i^{(1)})$ is the $(a \times 1)$ gradient vector evaluated at $\mathbf{x}_i^{(1)}$.
Dividing the jth row of the system (11) by the lth row, $j, l = 1, ..., a$, $j \neq l$, returns the well-known condition:

$$\frac{p_{ij}^{(1)}}{p_{il}^{(1)}} = \frac{\frac{\partial f(\mathbf{x}_i^{(1)})}{\partial x_{ij}^{(1)}}}{\frac{\partial f(\mathbf{x}_i^{(1)})}{\partial x_{il}^{(1)}}} \tag{22}$$

Table 1: Goods and assets entering the analysis

	Consumption goods
DUR	Durable Goods
NDUR	Nondurable Goods
SER	Services
	Leisure
LEIS	Leisure
	Monetary Assets
CP	Commercial Paper
CURTC	Currency and travelers checks
DD	Demand Deposits at commercial institutions
IMMF	Institutional Money market Funds
LTD	Large Time Deposits
MMDA	Money Market Demand Accounts at thrift and commercial institutions
OCD	Other Checkable Deposits at thrift and commercial institutions
SAV	Savings Accounts at thrift and commercial institutions
SAVM	Savings and Money Market Demand Accounts at thrift and commerical institutions
STD	Small Time Deposits at thrift and commercial institutions
REPO	Overnight and Term Repurchase Agreements
RMF	Retail Money Market Funds
TB	Short Term (3-Month) Treasury Bills

Hence, testing for weak separability amounts to testing for the independence between uniquely defined price ratios of goods inside the group (in log-form), and (log) quantities outside the group. To perform this task, we use the multivariate independence test introduced by El Himdi and Roy (1997), checking for noncorrelation between the innovations of two multivariate series, computed as residuals of two independent Vector Auto Regressive (VAR) processes. Let $\mathbf{ps}_i = \log(p_{i1}^{(1)}/p_{i2}^{(1)}, p_{i1}^{(1)}/p_{i3}^{(1)} ..., p_{i(a-1)}^{(1)}/p_{ia}^{(1)})'$ be the vector of unique price ratios of the separable group. Then the two VAR representations are given by:

$$\mathbf{b}_i^{(1)} = \mathbf{ps}_i' - \sum_{r=1}^{p_1} \mathbf{\Phi}_r \mathbf{ps}_{i-r}' \tag{23}$$

$$\mathbf{b}_i^{(2)} = \log(\mathbf{x}_i^{(2)})' - \sum_{r=1}^{p_2} \mathbf{\Psi}_r \log(\mathbf{x}_{i-1}^{(2)})' \tag{24}$$

Now, define R_{11} and R_{22} as the correlations matrices for $\mathbf{b}_i^{(1)}$ and $\mathbf{b}_i^{(2)}$, and R_{12} as the cross-correlations between $\mathbf{b}_i^{(1)}$ and $\mathbf{b}_i^{(2)}$, then the test statistic for noncorrelation is given by:

$$Qh = Tvec(R_{12})'(R_{11} \otimes R_{22})^{-1} vec(R_{12}) \tag{25}$$

Table 2: Goods and assets entering the analysis

	Periods				
	Period 1 Jan67-Dec69	Period 2 Jan70-May74	Period 3 Jun74-Nov82	Period 4 Dec82-Aug91	Period 5 Sep91-Dec12
Consumption goods					
DUR	Yes	Yes	Yes	Yes	Yes
NDUR	Yes	Yes	Yes	Yes	Yes
SER	Yes	Yes	Yes	Yes	Yes
Leisure					
LEIS	Yes	Yes	Yes	Yes	Yes
Monetary assets					
CURTC	Yes	Yes	Yes	Yes	Yes
CP	Yes	Yes	Yes	Yes	Yes
DD	Yes	Yes	Yes	Yes	Yes
OCD	Yes	Yes	Yes	Yes	Yes
STD	Yes	Yes	Yes	Yes	Yes
LTD	Yes	Yes	Yes	Yes	Yes
TB	Yes	Yes	Yes	Yes	Yes
SAV	Yes	Yes	Yes	Yes	No
REPO	No	Yes	Yes	Yes	Yes
RMF	No	No	Yes	Yes	Yes
IMMF	No	No	Yes	Yes	Yes
MMDA	No	No	No	Yes	No
SAVM	No	No	No	No	Yes

Under the null, Qh is distributed as a Chi-square with $d_1 d_2$ degrees of freedom, d_1 and d_2 being respectively the dimensions of $\mathbf{ps}_i$ and $\mathbf{x}_i^{(2)}$. Interestingly, the test can deal with stationary and nonstationary variables (possibly cointegrated).

We next turn to the empirical application of this methodology on US monetary and consumer data.

3 Implementing the tests

3.1 Data description

The test is applied to monthly per capita[6] data for consumption, leisure, and money in the United States from January 1967 to December 2012 (see Table 1). Data for consumption goods and services are taken from the Federal Reserve Bank of Saint Louis. Nondurable goods and

[6]Note that our test is therefore a joint test about utility maximization/weak separability and the existence of a representative agent.

Table 3: Monetary aggregates components

	DM1	DM2M	DM2 ALL	DM3	DM4-	DM4
CURTC	X	X	X	X	X	X
DD	X	X	X	X	X	X
OCD	X	X	X	X	X	X
SAV		X	X	X	X	X
SAVM		X	X	X	X	X
MMDA		X	X	X	X	X
RMF		X	X	X	X	X
STD			X	X	X	X
IMMF			X	X	X	X
LTD				X	X	X
REPO				X	X	X
CP					X	X
TB						X

services are real quantities. An associated price index is used.

For durables, a stock is constructed from current expenditures, as well as an user cost or rental price, following Diewert (1974) and Patterson (1991). The stock is computed as:

$$S_i = I_i + (1-\delta)S_{i-1}, i = 2, ..., T, S_1 = I_1 \tag{26}$$

where:

S_i is the stock in period i,

I_i in the expenditure in period i,

δ is a constant depreciation rate, set to $\delta = 10\%$.

and rental price at time i is defined as:

$$p_i = \frac{p_i^*[r_i^B + \delta_i(1+\Pi_i) - \Pi_i]}{\left(1 + r_i^B\right)} \tag{27}$$

where:

p_i^* is the spot price (index) for durable goods,

δ is the depreciation rate,

r_i^B is a benchmark rate,

Π_i is the period inflation rate for durable goods.

To build the leisure variable, we use nonmarket time, following Swofford and Whitney (1987). Nonmarket time is 98 hours less the average hours worked by nonsupervisory employees in the United States, multiplied by four to get monthly observations. The price of leisure is a shadow cost, as outlined by Barnett (1979a). The observed wage rate, in this case the average hourly earnings of nonsupervisory employees, does not take into account the effect on the price of leisure of employment in the economy. For full employment, the shadow price of leisure would equal the wage rate, however in times of higher unemployment the price of leisure would not be the same as the wage rate. For some observed wage rate w_i and level of employment E_i the shadow cost of leisure is defined as:

$$\hat{w}_i = w_i E_i^{\alpha} \tag{28}$$

where:
α is a parameter, set here to $\alpha = 2.3$, according to Barnett (1979a)

Monetary assets and user costs are taken from the data set provided on the Advances in Monetary and Financial Measures website, from the Center for Financial Stabilty (CFS). These data are collected from a variety of accessible resources described in detail in Barnett et al (2013). Real quantities are the assets divided by a consumer price index, whereas nominal user cost is defined according to Barnett (1978):

$$\pi_{ij} = p_i^{cpi} \frac{R_i^B - R_{ij}}{R_i^B} \tag{29}$$

where:
$R_i^B = 1 + r_i^B$, and r_i^B is the rate of a benchmark asset in period i,
$R_{ij} = 1 + r_{ij}$, with r_{ij} the asset j own rate in period i,
p_i^{cpi} is a consumer price index.
We use here the methodology of the CFS[7], where the benchmark rate is chosen as a short term rate for bank loans to commercial and industrial customers. This rate approximates the highest possible return that a bank would provide as the bank would not offer to pay more on interest than it loans out. Using this method described in Offenbacher and Schachar (2011) for the Bank of Israel, avoids the use of an arbitrary addition of basis points to a highest rate of return chosen from the basket of monetary asset returns[8].

Some components entered into the survey at later times, and others left the survey; for example Money Market Demand Accounts (MMDA) became part of the survey in the early 1980s, and were then placed into an overall measure of savings deposits and MMDA in the early 1990s. Also, paper currency and travelers checks are combined into one component ‘Currency

[7] See Table (3) for the definition of the monetary aggregates.
[8] See also Anderson and Jones (2011).

Table 4: Deterministic and stochastic tests for utility maximization

	Period 1 Jan67-Dec69	Period 2 Jan70-May74	Period 3 Jun74-Nov82	Period 4 Dec82-Aug91	Period 5 Sep91-Dec12
			Deterministic Analysis		
# of GARP violations	0	0	2	30	367
			Stochastic Analysis		
# of altered bundles	0	0	1	7	43
Obj			0.1690	2140.07	1128679.40
$Min(Obj_i)$(obs)			0.1690 (9)	0.0005 (86)	0.0240 (234)
$Max(Obj_i)$ (obs)			0.1690 (9)	2136.51 (100)	319093.75 (210)
Significance[1] at 5%	No	No	No	No	Yes

[1]: Test based on the empirical quantiles of the extremal smoothed errors, computed using the simulation smoother.

and Travelers Checks' due to the consistent and precipitous drop in the use of Travelers Checks. The differing thrift and commercial bank series are also combined and an average of their rates of return used to determine the nominal user cost. Savings, interest checking, MMDA, and small denomination time deposits for commercials and thrifts are therefore combined into series for savings, interest checking, MMDA, and small time deposits. For econometric analysis, the overall sample is then split into five periods (see Table 2). At last, note that all data are seasonally adjusted, (see Barnett et al 2013).

3.2 Testing for overall utility maximization

Since weak-separability is a rewriting of the overall utility function, we begin by testing for overall utility maximization (Condition 1 of the Varian procedure). We implement both the GARP and the de Peretti (2005) tests on a utility function in which enter goods, leisure and monetary assets.

Table (4) presents the results of the analysis. Focusing on the deterministic GARP, for periods 1 and 2, utility maximization is not rejected with a zero violation rate, whereas it is for periods 3, 4 and 5. While it seems not severe for period 3 (2 violations), periods 4 and 5 exhibit more violations (30 and 367). In order to draw some information about the distributions of violations, we use a graphical analysis of the violations of GARP for periods 4 and 5. Panel (1) presents four key graphs. The two upper figures are scatter plots of all couples (i, j) such that $\mathbf{x}_i R \mathbf{x}_j$ and $\mathbf{x}_j P \mathbf{x}_i$, that is all the observations which violate GARP. The two lower figures exhibit with how many observations, an observation i violates GARP.

Two interesting features arise. Violations are concentrated in the upper right quadrant $(i, j) \in$

Table 5: Results of the Barnett and de Peretti (2009) procedure for weak separability, periods 1, 2 and 3.

	DM1	DM2M	DM2 ALL	DM3	DM4-	DM4
			Period 1: Jan67-Dec69			
			Analysis for the sub-utility			
GARP	0	0	5	0	0	2
Adjustment	0	0	0.2631	0	0	0.0006
			Analysis for the overall utility and the sub-utility			
GARP	0	0	0	0	0	0
Overall adjustment	0	0	0.2631	0	0	0.0006
Significance[1] (5%)	.	.	No	.	.	No
			Separability Tests			
Independence[2]	3.27 (0.952)	45.77 (0)	52.12 (0)	76.76 (0)	98.14 (0)	90.46 (0)
Separability	Yes	No	No	No	No	No
			Period 2: Jan70-May74			
			Analysis for the sub-utility			
GARP	2	2	47	33	22	29
Adjustment	0.0003	0.3309	56082.84	5155.21	6055.98	4193.57
			Analysis for the overall utility and the sub-utility			
			Significance of the Adjustment			
GARP	0	0	0	0	0	0
Overall adjustment	0.0003	0.3309	56082.84	5155.21	6055.98	4193.57
Significance[1] (5%)	No	No	Yes	No	No	No
			Separability Tests			
Independence[2]	8.10 (0.618)	100.36 (0)	.	152.18 (0)	152.74 (0)	145.43 (0)
Separability	Yes	No	No	No	No	No
			Period 3: Jun74-Nov82			
			Analysis for the sub-utility			
GARP	7035	937	N.R.	2131	2193	2554
Adjustment	No Sol.	4748201.70	N.R.	3369718.00	3549231.00	9887286.31
			Analysis for the overall utility and the sub-utility			
GARP	.	6	N.R.	6	4	0
Overall adjustment	No Sol.	4698628.81	N.R.	3369718.20	3549232.80	9887286.31
Significance[1] (5%)	.	Yes	N.R.	Yes	Yes	Yes
			Separability Tests			
Independence[2]	.	.	N.R.	.	.	.
Separability	No	No	N.R.	No	No	No

[1]: Test based on the empirical quantiles of the extremal smoothed errors, computed using the simulation smoother.

[2]: Test based on 25.

N.R. stands for Not Relevant.

No.Sol. stands for No Solution to the quadratic program (8).

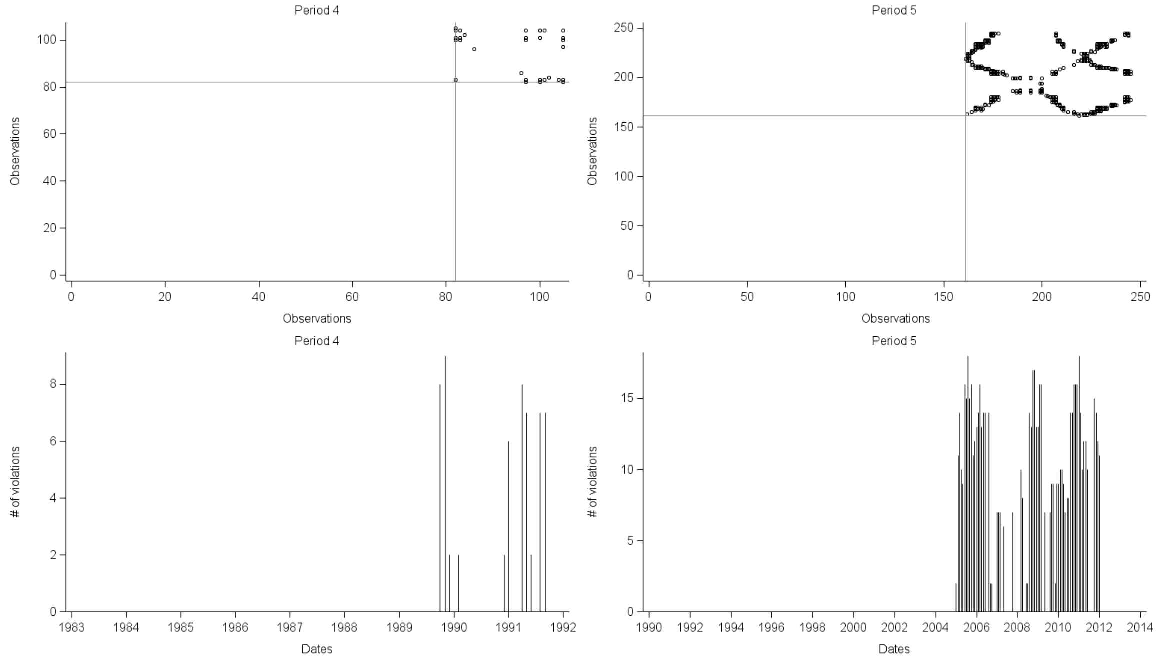

Figure 1: Distribution of violations. The two upper graphs return all couples (i, j) such that $\mathbf{x}_i R \mathbf{x}_j$ and $\mathbf{x}_j P \mathbf{x}_i$. The two lower graphs give with how many bundles a bundle $\mathbf{x}_i$ is violating GARP.

$(82, 105)$ (i.e. Sep89-Aug91) for period 4, and $i, j \in (161, 245)$ for period 5 (i.e. Jan05-Dec12). Concerning the two lower graphs, the problem with period 5 appears more severe than period 4, leading to doubt for utility maximization. For period 5, the high violation rate is in our point of view directly related to the sub-prime crisis. Either the crisis has caused ruptures in the utility function, or the 'easy money' period caused the agents to behave irrationally regarding money. We hypothesize that this latter argument is likely to be the cause of the violations. The first entries of Table (7) presenting the GARP analysis over monetary goods anticipate the overall analysis. For DM2 ALL, there is a high violation rate, indicating rejection of the null of utility maximization for period 5. This is likely to be caused by the earlier proposed lack of rationality over monetary assets.

Turning to the significance tests, the lower part of Table (4) presents the results of the de Peretti (2005) tests. Main entries are the number of bundles altered so that the whole set complies with utility maximization, the overall adjustment (obj), the minimal and maximal overall adjustments ($\max(obj_i)$ and $\min(obj_i)$), and a significance test[9]. Recall the procedure first filters x_{ij} for a given j, then the simulation smoother (20) is used a number of times to produce smoothed measurement errors. At each iteration, the two extrema (min and max) are stored and the empirical quantiles at 90%, 95% and 99% are computed. These quantiles are compared to the extremal theoretical adjustments for all goods, when the adjustments are computed using (6). If the extremal theoretical adjustment for one good (excess adjustment) exceeds the previously computed quantile, we reject the null of utility maximization. Note that the objective function presented in Table (4) depends on both the magnitude of the adjustment and on the number of bundles adjusted.

For period 3, with an overall (quadratic) adjustment of 0.1690 the null of utility maximization not rejected. A similar conclusion holds for period 4 (where only 7 bundles are altered), but not for period 5, with a massive adjustment of 1128679.40, and 43 bundles altered. Thus the overall utility maximization assumption holds for periods 1, 2, 3 and 4 but not for period 5. For this latter period, the large difference between the minimal and the maximal overall adjustments for some period i, as well as the standard error of obj_i (63870.55), indicates a high dispersion among the adjustments; suggesting that some violations may be caused by measurement errors and others by ruptures in the utility function, or random behavior.

Since violations of GARP are linked by transitive relationships, finding sub-samples in which GARP is not violated, or when violations are nonsignificant is no easy task. Assuming a single rupture in the utility function, we split Period 5 into two sub-periods $(1, int(\lambda T_5))$ and $(int(\lambda T_5)+1, T_5)$, where T_5 is the number of observations in Period 5 and $\lambda \in (0.1, 0.9)$. Using a search algorithm over λ, we identify two sub-samples $(1-198)$ and $(207-254)$ corresponding to Sep1991-Feb2008 and Jan2009-Dec2012, on which GARP is violated, but the violations are nonsignificant.

In what follows, weak separability tests are implemented for periods 1, 2, 3 and 4, and on periods 4', Dec82-Sep1989 (period with no violations of GARP), periods 5', 5" and 5"', corresponding to respectively to Sep91-Feb05 (No violation of GARP), Sep1991-Feb2008 and Jan2009-Dec2012.

3.3 Testing for weak separability

Turning now to weak separability testing, we test six different aggregates DM1, DM2M, DM2 ALL, DM3, DM4- and DM4 as defined by the CFS, for the periods defined (see Table 3). Tables

[9] To avoid any confusion, we use here a significance test based on the simulation smoother, not on the iid properties of the adjustments, as introduced in de Peretti (2005).

Table 6: Results of the Barnett and de Peretti (2009) procedure for weak separability. Periods 4 and 4'

	DM1	DM2M	DM2 ALL	DM3	DM4-	DM4
Period 4: Dec82-Aug91						
Analysis for the sub-utility						
GARP	218	62	47	4	23	27
Adjustment	14006.75	2753.53	413.91	0.012	607.13	679.69
Analysis for the overall utility and the sub-utility						
GARP	31	32	30	30	30	30
Overall adjustment	14010.643	5238.98	3091.48	2540.63	3417.64	8342.94
Significance[1] (5%)	No	No	No	No	No	No
Separability Tests						
Independence[2]	15.01 (0.306)	395.40 (0)	372.16 (0)	380.27 (0)	328.28 (0)	274.86 (0)
Separability	Yes	No	No	No	No	No
Period 4': Dec82-Sep1989						
Analysis for the sub-utility						
GARP	84	44	45	2	10	18
Adjustment	5714.87	1724.66	413.90	0.0001	48.67	644.99
Analysis for the overall utility and the sub-utility						
GARP	0	2	0	0	0	0
Overall Adjustment	5714.87	1447.27	413.90	0.0001	48.67	644.99
Significance[1] (5%)	No	No	No	No	No	No
Separability Tests						
Independence[2]	13.61 (0.401)	312.46 (0)	319.66 (0)	314.67 (0)	266.10 (0)	230.53 (0)
Separability	Yes	No	No	No	No	No

[1]: Test based on the empirical quantiles of the extremal smoothed errors, computed using the simulation smoother.
[2]: Test based on (25).

(5) to (7) present the results: The main entries being: i) For the sub-utility, the number of violations of GARP, as well as the theoretical adjustment if violations appear, ii) For both the overall utility and the sub-utility the same information plus a significance test based on the overall adjustment, iii) For the separability test, the value of Qh and the p-value. Recall a sub-utility (Condition 2) is first tested, and if violations appear the data are adjusted. Then we test GARP on the overall data set, with possible partially adjusted data. If violations appear, we produce a set compliant with both the utility and the sub-utility maximization programs (Condition 1 given that Condition 2 holds), as explained previously. We then perform significance tests on the overall adjustment and if violations are nonsignificant, we use the separability test[10].

[10] This non correlation test is assumed to be robust to additive outliers. We implement the tests on both adjusted and non-adjusted data. The results of the tests did return the same information. In this article, only

Table 7: Results of the Barnett and de Peretti (2009) procedure for weak separability. Periods $5'$, $5''$ and $5'''$

	DM1	DM2M	DM2 ALL	DM3	DM4-	DM4
			Periods $5'$: Sep91-Feb05			
			Analysis for the sub-utility			
GARP	6	47	245	10	46	17
Adjustment	1.17	11550.95	113044.28	166.48	5060.95	194.62
			Analysis for the overall utility and the sub-utility			
GARP	0	0	0	0	0	0
Overall adjustment	1.17	11550.95	113044.28	166.48	5060.95	194.62
Significance[1] (5%)	No	No	No	No	No	No
			Separability Tests			
Independence[2]	14.57 (0.265)	484.65 (0)	545.83 (0)	681.63 (0)	463.52 (0)	389.36 (0)
Separability	Yes	No	No	No	No	No
			Period $5''$: Sep91-Feb08			
			Analysis for the sub-utility			
GARP	6	797	2174	709	770	714
Adjustment	1.17	274085.51	1629798.30	710457.29	944018.13	537723.51
			Analysis for the overall utility and the sub-utility			
GARP	0	0	8	2	6	4
Overall adjustment	1.17	274085.51	1629798.30	710457.29	943954.36	538734.75
Significance[1] (5%)	No	Yes	Yes	Yes	Yes	Yes
			Separability Tests			
Independence[2]	22.84 (0.029)	.	.	.	.	.
Separability	No	No	No	No	No	No
			Period $5'''$: Jan09-Dec12			
			Analysis for the sub-utility			
GARP	0	9	0	0	0	0
Adjustment	0	12.48	0	0	0	0
			Analysis for the overall utility and the sub-utility			
GARP	19	19	19	7	19	19
Overall adjustment	1500.78	1509.78	1575.44	7.59	1497.29	1550.16
Significance[1] (5%)	No	No	No	No	No	No
			Separability Tests			
Independence[2]	16.91 (0.152)	144.92 (0)	157.52 (0)	202.27 (0)	167.18 (0)	146.42 (0)
Separability	Yes	No	No	No	No	No

[1]: Test based on the empirical quantiles of the extremal smoothed errors, computed using the simulation smoother.
[2]: Test based on (25).

For period 1 (Jan67-Dec69), Table (5), when testing for GARP on the sub-utility, only two

the results for the non-adjusted data are presented.

clusters present violations: DM2 ALL and DM4. Nevertheless, the required adjustment is low (0.2631 and 0.0006), and nonsignificant. When testing for separability, we found that only the DM1 group is separable, while for all other aggregates, separability is strongly rejected. The analysis is more interesting for period 2 (Jan70-May74) since for all monetary clusters, violations appear and except for DM2 ALL, the theoretical adjustment is nonsignificant. Again, DM1 is the only aggregate found out separable. Note that when monetary data are adjusted, no violation occur when testing for the overall utility maximization program. Testing for a sub-utility function and an overall utility function for period 3 produces a high number of violations for all clusters with huge and significant adjustments. For DM1 we were not even able to solve (8). Compared to the results we obtained when testing for overall separability (only 2 violations), we here detect a high instability in monetary behaviors, leading to rejection of weak separability.

We now turn to periods 4 and 5 (Tables 6 and 7) that reveal a large number of violations when testing for the overall utility maximization program. For the former period, we perform the tests on both the overall sample (Dec82-Aug91), and on the period when no violation appeared (Dec82-Sep89). On Dec82-Aug91, the analysis produces GARP violations for both the monetary utility functions and the overall utility function, as suggested in the previous section. Nevertheless, the adjustments are non significant, even, surprisingly when testing for DM1 (total adjustment: 14006.75). Here again, only DM1 appears to be weakly separable. A similar conclusion is drawn when focusing on period 4', except that less violations are found.

For the last period, we perform the analysis over three sub-periods, Sep91-Feb05, where no violations appear when testing for the overall maximization, Sep91-Feb08 and Jan09-Dec12, the latter two sub-periods presenting no significant violations of GARP. For Sep91-Feb05, there exists both a sub-utility function and an overall utility function for all clusters, even if the adjustments seem large for DM2M and DM2ALL, but only DM1 is weakly separable. On Sep91-Feb08, the assumption of a sub-utility function rationalizing the data is strongly rejected for all clusters, except DM1, which is not separable at the 5% level, confirming high instability of monetary behaviors during the sub-prime crisis. Lastly, during Jan09-Dec12, still only DM1 presents empirical evidence of weak separability.

4 Conclusion and discussion

This paper is an attempt to provide researchers in monetary economics with sound basis for choosing monetary aggregates. We have therefore tested six different aggregates for weak separability using stochastic semi-nonparametric tests as defined by Barnett and de Peretti (2009). The tests are based on a different Condition 3. With regard to the classical Varian's approach it uses different information based on the independence between the marginal rate of substitution of assets inside the group, and goods outside the cluster. Moreover, it allows the researcher to test the significance of the departure from the two necessary maximization problems: utility and sub-utility.

Our paper emphasizes important results having key implications regarding monetary economics. The results are twofold.

i) Concerning separability, it supports a narrow monetary aggregate. Indeed, the DM1 cluster is the only one found separable over different time periods. There is no evidence provided for broader aggregate separability.

ii) Tests over recent and crisis related periods such as June 1974 to November 1982 and September 1991 to December 2012 detect high instability in monetary behaviors. For instance, during the stagflation and savings and loan crisis period of 1974 to 1982, we were unable to find monetary sub-utility functions rationalizing the data. Concerning the Great Recession period, instability was both in the overall utility and in the monetary sub-utility functions. Looking at the theoretical adjustments reveals a high disparity: Some bundles require a small adjustment to be compliant with GARP, while other a large one. This is in our point of view suggestive of both optimization errors and of some ruptures in the (sub) utility functions. We therefore hypothesize that the observed non-rationality is directly related to crisis. This point requires further investigations, especially about the direction of causality.

In this paper we have tested for some clusters widely used in monetary economics. We show that even if a monetary utility does exist there is little evidence of its weak separability, except for DM1. Nevertheless, since for testing for separability, we use a multivariate independence test, it could be very easy, looking at the cross correlations matrices, to deduce a monetary aggregate. It also could well be the case, since the CFS uses a certainty equivalent user cost calculation, that the violation rate observed during crisis periods might be caused by the inability of the monetary asset user cost to capture in the effect of risk in broad components such as overnight repurchases and commercial paper. If this is the case, then a risk adjustment is necessary for the CFS data along the lines of Barnett and Wu (2005). Both possibilities provide avenues for further research into weak separability testing.

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