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Unobserved components in corporate defaults and bond prices*

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Abstract

If there is an unobserved component in corporate default intensities, then part of the fluctuation in corporate bond prices can be attributed to the variation in beliefs about this latent factor over time. Using sequential Markov Chain Monte Carlo techniques, we show evidence of a latent frailty process in the default intensities in U.S. corporate defaults. The factor is robust to the inclusion of both macro and firm specific variables. We use the sequentially estimated changes in conditional expectations of the frailty level, persistence and volatility to proxy for changes in agents' beliefs about the unobserved default intensities. We find that changes in frailty related variables help to explain the variation in U.S. corporate credit spreads and increase explanatory power by 10%–22%.

Keywords: unobserved components, corporate bonds, credit risk, Bayesian inference.

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1 Introduction

There is a large body of literature trying to explain the sources of default clustering present in historical default data. However the literature is still not conclusive about whether defaults are independent condition on observable factors or whether there is a separate independent default risk "frailty" factor; see Duffie, Eckner, Horel, and Saita (2009) and Lando and Nielsen (2010). This question is important because it is essential for prudent credit risk management and it has crucial asset pricing implications. If default risk depends on an unobservable factor agents should infer the level and dynamics of the latent factor from the available information and incorporate this in asset prices.

Our main contribution in this paper is to demonstrate that part of the time series variation in corporate credit spread can be attributed to an unobserved component in default intensities, even after accounting for both macro-wide and firm-specific observed control variables. First we contribute to the credit risk literature by providing further evidence on an unobserved factor in corporate default intensities. Moreover we show that agents' beliefs about the level of this latent factor negatively correlated with corporate credit spreads.

In the first part of the paper we revisit the question whether defaults are independent conditional on observable covariates. There appears to be no consensus about this in the literature. Das, Duffie, Kapadia, and Saita (2007) and Duffie et al. (2009) argue that we need a latent, so called frailty process to explain the default clustering present in the data, while Lando and Nielsen (2010) cannot reject the hypothesis that the defaults are independent conditional on several firm specific and macro variables. We contribute to this discussion by estimating a dynamic credit risk model using US corporate defaults from January 1980 till March 2010 and including frailty as well as the firm specific variables used by Lando and Nielsen (2010). Based on the marginal likelihoods of different specifications, we find that even after accounting for the firm specific and macro variables suggested by Lando and Nielsen (2010) the model with frailty is supported by the data.

In the second part of the paper we focus on the link between corporate credit spreads and the unobserved frailty factor. We find that a one unit decrease in the agent's belief about the level of frailty relates to approximately a 0.6% increase in the credit spread on average. In order to obtain this result, we first construct a proxy for agents' beliefs about the level of frailty. We assume Bayesian agents with a quadratic loss function who try to infer the default intensity based on the observed defaults and firm specific and macro variables. Observing firm specific and macro fundamentals, they sequentially update their beliefs about the parameters and frailty factor. We numerically solve this filtering problem and obtain an estimate of the frailty at time t given all information up until time t. To test if frailty effects corporate credit spreads we run a panel regression of corporate credit spreads on our previously obtained proxy for agents' beliefs, credit risk and liquidity controls and bond specific control variables using monthly data from October 2004 till March 2010. We find that frailty helps to explain the variation in credit spreads and that the effect of learning is larger for speculative grade bonds.

This paper is related to existing literature on Bayesian learning in asset pricing. The early literature is concerned about the effect of unobservable economic states on returns (Detemple (1986) and David (1997)) and the implication of learning about growth rates on stock prices (Pastor and Veronesi (2003) and Pastor and Veronesi (2006)). In the credit risk context Collin-Dufresne et al. (2010) propose a tractable pricing model where agents learn from the observed defaults history. Recently Collin-Dufresne et al. (2013) and Johannes et al. (2013) find that learning generates large shocks to beliefs about long run consumption. Agents which preference for early resolution of uncertainty require compensation for this. Pastor and Veronesi (2009) give an overview about learning models in asset pricing.

Our work is also related to the corporate bond pricing literature, e.g. Driessen (2005), Houweling et al. (2005), Dick-Nielsen et al. (2012). These articles try to quantify liquidity premium in corporate bond prices. Although, our focus is not on the liquidity premium we control for the effect of liquidity on corporate bond prices. Finally we also use results from the credit risk literature e.g. Das et al. (2007), Duffie et al. (2009), and Koopman et al. (2008). These papers are concerned with the latent frailty process in corporate default intensities. In our paper we use these results, but we study the pricing implications of the frailty process.

The rest of the paper is organized as follows. In the next section we use a simple example to show what happens with corporate bond prices if the default intensity is unknown to agents. Section 3 describes the estimated dynamic credit risk models and the sequential estimation procedure. Section 4 elaborates on the data we used. Section 5 presents our findings about the unobserved component. Finally Section 6 explains the empirical link between corporate credit spreads and frailty.

2 A simple motivating example

In this section we show why it is important from an asset pricing perspective to know if the default intensity depends on an unobservable factor. We use a simple example to show that bond prices can vary over time when agents do not know the default intensity. This is true even if the default intensity and risk free rate are constant and agents are risk neutral.

Assume there is a defaultable zero-coupon bond with zero recovery and maturity T. Moreover assume that the risk free rate r_f is constant and that the default happens at a random stopping time τ with constant intensity λ. In this case the price of the bond $B(t,T)$ at time t is given by

$$B(t,T) = \mathrm{E}_t\left(\mathbb{1}_{\{\tau>T\}}\right) D(t,T) + \mathrm{Cov}_t\left(M_{t,T}, \mathbb{1}_{\{\tau>T\}}\right), \tag{1}$$

where $M_{t,T}$ is the stochastic discount factor between t and T and $D(t,T) = \exp\left[-r_f(T-t)\right]$. Assume that λ is known and constant and that agents are risk neutral. This leads to the following formula

$$B(t,T) = \mathrm{E}_t\left(\mathbb{1}_{\{\tau>T\}}\right) D(t,T) = \exp\left[-\lambda(T-t)\right] D(t,T) = \exp\left[-(r_f+\lambda)(T-t)\right]. \tag{2}$$

We now relax the assumption that the intensity is known and instead assume that the agents observe monthly corporate default counts from which they try to infer the constant default intensity. We denote agents information set at time t as $\mathcal{F}_t$ which contains the observed number of firms n_i and defaults d_i until time t $i = 1,\dots,t$. Using the anticipated utility assumption (see e.g. Kreps (1998) and Cogley and Sargent (2008)), which says that agents learn about parameters, but they treat them constant when they make a decision, we obtain

$$\begin{aligned}
B(t,T) &= \mathrm{E}_t\left(\mathbb{1}_{\{\tau>T\}}|\mathcal{F}_t\right) D(t,T) \\
&= \mathrm{E}_t\left[\mathrm{E}_t\left(\mathbb{1}_{\{\tau>T\}}|\lambda,\mathcal{F}_t\right)|\mathcal{F}_t\right] D(t,T) \\
&= D(t,T)\int \mathrm{E}_t\left(1_{\{\tau>T\}}|\lambda\right) p\left(\lambda|\mathcal{F}_t\right) \mathrm{d}\lambda \\
&= D(t,T)\int \exp\left[-\lambda(T-t)\right] p\left(\lambda|\mathcal{F}_t\right) \mathrm{d}\lambda.
\end{aligned} \tag{3}$$

where $p\left(\lambda|\mathcal{F}_t\right)$ is the posterior density of λ. Using a first order Taylor series approximation we obtain

$$
\begin{aligned}
B(t,T) &\approx D(t,T)\left[1-(T-t)\int \lambda p\left(\lambda|\mathcal{F}_t\right)\mathrm{d}\lambda\right] \\
&= D(t,T)\left[1-(T-t)\mathrm{E}_t(\lambda|\mathcal{F}_t)\right],
\end{aligned}
\tag{4}
$$

where $\mathrm{E}_t(\lambda|\mathcal{F}_t)$ is the posterior mean of the intensity.

Using the fact that the probability of default over Δt is approximately $\lambda\Delta t$, the probability of observing d_i defaults from a homogeneous portfolio of n_i firms over period i is given by

$$
\binom{n_i}{d_i}\left(\lambda\Delta t\right)^{d_i}\left(1-\lambda\Delta t\right)^{n_i-d_i}. \tag{5}
$$

Using a conjugate beta prior $\mathrm{Beta}(\alpha_0,\beta_0)$ for the probability of defaults with density function given by

$$
p(x;\alpha_0,\beta_0)=\frac{1}{\mathrm{B}(\alpha_0,\beta_0)}x^{\alpha_0-1}(1-x)^{\beta_0-1}, \tag{6}
$$

the posterior mean of λ given the observations n_i and k_i $i=1\dots t$ is

$$
\mathrm{E}_t(\lambda\Delta t|\mathcal{F}_t)=\frac{\alpha_t}{\alpha_t+\beta_t}=\frac{\alpha_0+\sum_{i=1}^{t}d_i}{\alpha_0+\beta_0+\sum_{i=1}^{t}n_i}. \tag{7}
$$

Substituting (7) into (4) yields the following expression for the bond price

$$
B_t\approx D(t,T)-D(t,T)\frac{(T-t)}{\Delta t}\frac{\alpha_0+\sum_{i=1}^{t}d_i}{\alpha_0+\beta_0+\sum_{i=1}^{t}n_i}, \tag{8}
$$

which shows that the bond price varies over time even if the default intensity is constant and the interest rates are constant just because agents do not know the intensity parameter and have to estimate it using available data. This shows that it is important to know whether default intensities depend on an unobservable factor or not. If they do, then agents also have to infer its value basedon available data at each time t and their estimate impacts asset prices.

Note that in our simple example, we assume that the intensity is fixed. This means that learning about the intensity eventually dies out after accumulating enough data. However in case of an unobserved process agents continuously have to update their beliefs about the current state of the latent factor.

Also note that to derive the bond price in our above example we had to make simplifying assumptions and approximations. In general there is little hope that we can provide an analytic solution for the bond price as already the filtering problem does not have a closed form solution. Thus we resort to a numerical approximation of the filtering density and estimate a reduce form asset pricing model by regressing corporate credit spreads on the sequential posterior mean estimates of the unobservable process.

3 Modelling approach and estimation

In this section we introduce the dynamic credit risk model which is used later on to estimate the frailty factor. The goal is to predict default probabilities for any given time interval given the observed covariates. The first step is to specify the default intensity λ_{ti} of firm i at time t,which is defined as the normalized instantaneous default probability of firm i at time t given that firm i is alive at time t

$$\lambda_{ti} = \lim_{\Delta t \downarrow 0} \frac{P\left[t < T_i \leq t + \Delta t | T_i > t\right]}{\Delta t}, \tag{9}$$

where T_i is the default time of firm i. We model the default intensity for firm i at time t as a function of firm specific and macro fundamentals collected in a vector c_{it} and a latent frailty factor f_t

$$\begin{aligned} \log \lambda_{ti} &= \mu + \beta c_{it} + \gamma f_t, & (10)\\ f_{t+1} &= \phi f_t + \eta_{t+1}, \quad \eta_{t+1} \sim \mathrm{N}(0, 1-\phi^2), & (11)\\ f_1 &\sim \mathrm{N}(0,1), \end{aligned}$$

The likelihood of the observed defaults and survivals can be expressed as

$$\prod_{i=1}^{N_t} \exp\left(D_{it} \log \lambda_{it} - S_{it}\lambda_{it}\right), \tag{12}$$

where N_t is the number of firms in period t, D_{it} is a default indicator which is one if firm i defaults in period t and S_{it} is the time that firm i spent in the portfolio in period t.

Equation (10), (11) and (12) define a nonlinear, non-Gaussian state space model. Estimation of the model is challenging, because the likelihood is not available in closed form as the latent state has to be integrated out (see e.g. Durbin and Koopman (2012) and Chapter 2 of this thesis). As we would like to construct how Bayesian agents perceive frailty over time, we have to estimate the model sequentially given the data available time t, where $t = 1, \ldots, T$. Sequential estimation of the model is even more involved as we have to estimate the model for every time period.

Chopin et al. (2013) and Fülöp and Li (2013) propose a sequential estimation procedure for nonlinear, non-Gaussian state space models. The method can be thought of as the extension of the procedure suggested by Chopin (2002) for the case when the likelihood is not available in closed form, but can still be estimated with particle filters.

The algorithm is given by the following steps. Let $\theta^m = \{\beta^m, \gamma^m, \phi^m\}$ and $y_{1:t} = (D_{ij}, S_{ij}; i = 1, \ldots, N_j, j = 1, \ldots, t)$. Sample $\theta^m, m = 1, \ldots, N_\theta$, from the prior $p(\theta)$ and set $\omega^m = 1, m = 1, \ldots, N_\theta$. For time $t = 1, \ldots, T$ do the following steps.

1. (a) If $t = 1$ then sample $f_1^{1:N_f,m}$ from $\mathrm{N}(0,1)$, for $m = 1, \ldots, N_\theta$ and calculate

$$\hat{p}(y_1|\theta^m) = \sum_{n=1}^{N_f} \omega^{n,m} = \sum_{n=1}^{N_f} \prod_{i=1}^{N_1} \exp\left(D_{i1} \log \lambda_{i1}^{n,m} - S_{i1}\lambda_{i1}^{n,m}\right), \qquad (13)$$

where

$$\lambda_{i1}^{n,m} = \beta^m c_{it} + \gamma^m f_1^{n,m}. \qquad (14)$$

(b) If $t > 1$ then resample $f_{t-1}^{n,m}$ according to the normalized weights

$$W^{n,m} = \frac{\exp(\omega^{n,m})}{\sum\limits_{n=1}^{N_f} \exp(\omega^{n,m})} \qquad (15)$$

for $m = 1, \ldots, N_\theta$, $n = 1, \ldots, N_f$ to obtain the resampled states $\tilde{f}_{t-1}^{n,m}$ and propagate $f_{t-1}^{n,m}$ forward by sampling $f_t^{n,m}$ from $\mathrm{N}(\phi^m \tilde{f}_{t-1}^{n,m}, 1-(\phi^m)^2)$ and calculate

$$\hat{p}(y_t|y_{1:t-1};\theta^m) = \sum_{n=1}^{N_f} \prod_{i=1}^{N_t} \exp\left(D_{it} \log \lambda_{it}^{n,m} - S_{it}\lambda_{it}^{n,m}\right), \qquad (16)$$

where

$$\lambda_{it}^{n,m} = \beta^m c_{it} + \gamma^m f_t^{n,m}. \tag{17}$$

2. Update the importance weights

$$\omega^m = \omega^m \hat{p}(y_t|y_{1:t-1};\theta^m). \tag{18}$$

3. If $ESS < N_f/2$ then resample the parameters $\theta^m, m = 1, \dots, N_\theta$ with the normalized weights

$$W^m = \frac{\exp(\omega^m)}{\sum\limits_{m=1}^{N_\theta} \exp(\omega^m)} \tag{19}$$

to get the the resampled parameters $\tilde{\theta}^m, m = 1, \dots, N_\theta$. Finally carry out a move step by proposing new particles $\bar{\theta}^m, m = 1, \dots, N_\theta$ from a multivariate normal distribution with mean $\mu_g = W^m\theta^m$ and variance $\sigma_g^2 = \sum\limits_{m=1}^{N_\theta} W^m(\theta^m)^2 - (\sum\limits_{m=1}^{N_\theta} W^m\theta^m)^2$ and accept them with probability

$$\alpha = \min\left\{\frac{p(y_{1:t}|\bar{\theta}^m)p(\bar{\theta}^m)g(\tilde{\theta}^m)}{p(y_{1:t}|\tilde{\theta}^m)p(\tilde{\theta}^m)g(\bar{\theta}^m)}, 1\right\} \tag{20}$$

where g is a multivariate normal density with mean μ_g , and variance σ_g^2, $p(y_{1:t}|\theta)$ is the likelihood at θ and $p(\theta)$ is the prior density at θ. Let the accepted parameters be θ^m and set $\omega^m = 1$.

The particles and weights at time t give an approximation to the posterior distribution of the parameters and latent state given the available data up to time t in the sense that for any integrable function h

$$\mathrm{E}\left[h(\theta)|y_{1:t}\right] \approx \sum_{m=1}^{N_\theta} W^m h(\theta^m), \tag{21}$$

and

$$\mathrm{E}\left[h(f_t)|y_{1:t}\right] \approx \sum_{n=1}^{N_f}\sum_{m=1}^{N_\theta} W^{n,m} h(f^{n,m}). \tag{22}$$

This means that we can approximately solve the sequential estimation problem of a Bayesian agent, who tries to infer the latent frailty process from observed data.

4 Data

We use US corporate default data from 1st January 1980 till 4th March 2010 from Moody's to carry out the sequential estimation of the default intensity. We calculate default spells from the rating history by ignoring rating withdrawals. We disregard parental defaults identified by the the same default dates and parental id. After filtering on the available CUSIPs in Compustat we end up with 3 768 firms and 773 defaults.

We start from the Moody's data set and merge monthly stock returns and shares outstanding from the daily and monthly CRSP files and quarterly[1] accounting variables from Compustat based on CUSIPs. After filtering the data and dropping the firm months without available firm specific accounting variables we end up with 2732 different GVKEYs and 351 defaults.

We download the 3-month Treasury Bill rate and the spread between the 1 year Treasury rate and 10 year Treasury rate, the monthly industrial production growth rate from the St. Louis FED FRED database. Moreover we use trailing 1 year S&P500 returns from CRSP.

In the second step we use the TRACE data set from 1st October 2004 to 31st March 2010. The reason for using data from only 1st October 2004 is that only from that point it was obligatory to file reports about all the bond trades. We start with 39119613 trade reports for 41883 CUSIPs. We obtain obtain bond specific information from DataStream and SDC Platinum. After filtering for bullet bonds with fixed coupons and without any callable or convertible futures we are left with 22 794 bonds from DataStream and 14 235 bonds from SDC Platinum.

We merge the static bond information from DataStream and SDC Platinum with TRACE using ISIN and CUSIP codes and end up with 4161 bonds and 11692831 trade reports. We clean the remaining reports from duplicates, reversals and same day corrections as described in Dick-Nielsen (2009). After the cleaning we have 11013806 trade reports. Following Dick-Nielsen, Feldhutter, and Lando (2012) we delete the retail sized transactions defined as the transactions below 100 000 USD notional yielding to 2693533 remaining transactions. Removing the trade reports with negative yields results in 2693177 transactions. In the final step we merge CRSP and Compustat variables with the TRACE data set using company tickers and filter on non-financial firms according to DataStream. At the end we have 389

[1] If it was not available we use yearly figures.

firms and 1767 bonds for which we have both bond transaction data, static bond data and accounting variables. We construct the yield spread using daily yield curve points from DataStream and a Nelson Siegel approximation between the yield curve points.

We construct the distance to default by solving the nonlinear equation system which is derived from the fact that the Merton model implied equity volatility and equity price should match the observed equity price and volatility.

5 Corporate default dynamics and frailty

In this section we present the empirical findings of the first stage analysis. First we present the results of the sequential intensity estimation then we discuss the findings of our panel regression.

Before the estimation of the model given by (10, (11) and)(12), we have to specify the covariates c_{it} in the specification of the intensity (10). There is a huge body of literature dealing with dynamic credit risk models and possible covariates (see e.g. Duffie et al. (2009) and Lando and Nielsen (2010)). We can divide the commonly used variables into two groups, namely macro variables which are common to all firms and firm specific variables. Table 1 summarizes the covariates.

Table 1: Descriptive statistics of the firm specific and macro variables used in intensity model. The table show the number of observations (Obs), the mean of the variables (Mean), the standard deviation (Std. Dev) , the skewness (Skewness) and finally the kurtosis (Kurtosis).

	Obs	Mean	Std. Dev.	Skewness	Kurtosis
Common					
3-month Treasury Bill	363	5.39	3.26	0.77	3.72
Treasury spread	363	1.22	1.16	-0.34	3.10
Industrial production growth	363	0.00	0.01	-1.18	8.39
1-year S&P 500 returns	363	0.07	0.17	-1.00	4.35
Previous month default counts	363	2.12	2.21	1.24	4.00
Firm-specific					
1-year stock returns	303196	-0.03	0.49	-1.16	11.20
Distance to default	303196	3.80	3.46	-5.60	14.85
Quick ratio	303196	1.13	4.80	1.67	29.11
Short term debt	303196	0.15	0.18	2.04	7.79
Log asset value	303196	7.46	1.58	0.11	2.81

In the literature there is no consensus about the specification of the firm specific intensities. Duffie et al. (2009) uses the 3-month Treasury Bill, the 1-year S&P500

returns, 1-year stock return and the distance to default as explanatory variables besides the frailty process. They could not reject the model with the frailty process. Lando and Nielsen (2010) use a different specification and they include some additional explanatory variables: Treasury spread, industrial production growth, quick ratio, log asset value and the short term debt. Instead of a incorporating frailty in their specification they choose to include a contagion effect, but they find no evidence of contagion and do not reject the hypothesis that the intensities are well specified. In our specification we follow Lando and Nielsen (2010) and use a similar intensity specification except for the fact that we also include frailty and that we provide sequential estimation results for our second stage analysis in 6.

The 3-month Treasury bill is used as a covariate, because short rates usually are set low during macro economic distress, which also means that we expect a negative coefficient on this variable. The difference between the long end and the short end of the yield curve is negative when investors expect a recession. Hence the Treasury spread can be a useful covariate, and this reasoning suggests a negative coefficient on this variable. Industrial production growth is low during recessions which suggests a negative relationship between the default intensity and industrial production growth. The trailing S&P 500 return is also a proxy for the state of the US economy as it is high in boom periods and low in recessions. Finally the previous month default counts can capture possible contagion effects as a month with high default counts is usually followed by a month with above average defaults due to default clustering.

There are two important considerations which we have to take into account when we decide about which firm specific variables to include in the model. First of all we would like to put in as many variables as possible, because the omission of a relevant variable can lead to omitted variable biases. However including too many variables decreases the number of available firm months in the sample. We have decided to include the following variables following Lando and Nielsen (2010)).

One of the most important firm specific variable is the distance to default. We can think of distance to default as the volatility adjusted leverage and we expect a negative coefficient on this variable. Another important covariate is the trailing stock return, which usually has a significant negative coefficient (see e.g. Duffie et al. (2009) and Lando and Nielsen (2010)). The quick ratio is defined as the sum of cash, short term investments and receivables divided by current liabilities and is believed to capture the short term liquidity of the firm. A lower quick ratio is associated with higher default risk. The logarithm of assets is a proxy for the size of the company which is also an often used variable as bigger companies have more options to avoid bankruptcy; see Lando and Nielsen (2010)). Based on this observation we expect a negative coefficient on the logarithm of assets. Finally, short term debt calculated as a percentage of total debt is a useful funding liquidity measure. High percentage of short term debt is expected to increase the default intensity.

The frailty estimate depends on the precise specification of the model. To check the possibly different implications of different specifications we estimate the following models. We estimate a frailty model with macro covariates, a frailty model with macro covariates plus the median of firm specific variables, and finally a frailty model with macro and firm specific variables. As a benchmark we estimate these models also without frailty. As a by product of sequential Bayesian estimation, we also obtain the marginal likelihoods of all our models, which we can use for model comparison.

Using the different model specifications, we check what are the implications of including firm specific variables on frailty. By comparing the frailty process from the model with the median firm specific variables and the one for a model containing only macro variables, we can check if frailty only picks up the missing firm specific variables. In addition we can also check the implications of the firm specific intensities on the the frailty process by contrasting the frailty process from the model with median firm specific variables to the process from the model with firm specific variables.

Table 2 shows the posterior mean estimates of the intensity model at the end of the sample for the different specifications. In the pure intensity models without frailty zero is not in the 95% credible set for all the firm specific variables. Moreover from the macro variables the credible set of the Treasury spread and 1-year S&P500 return variables exclude zero. Finding zero in the credible set of the previous quarter default counts is consistent with the findings in Lando and Nielsen (2010), where they argue that contagion operates through firm specific covariates. Hence the previous default counts are not important after controlling for firms specific variables. The sign of the coefficients are as expected, except the coefficient of the Treasury spread and the 1-year S&P500 return. The positive loading on the 1-year S&P500 return is consistent with the findings of Duffie et al. (2009).

The frailty model with macro variables (iv) on the credible set of the previous quarter default counts does not contain zero. The sign of the coefficient of the

Treasury spread becomes smaller compared to the intensity model with macro and firm specific variables (iii). In fact, it becomes negative, but the credible set also contains zero.

The fifth model includes macro variables and the median of the firm specific variables as well, as well as a frailty factor. All credible sets of the median firm specific variables contain zero, except for the distance to default variable. The loadings on the macro variables are similar to the ones from the frailty model with macro variables (iv). However the magnitude of the frailty component as measured by the frailty loading is much smaller in this model due to the inclusion of the median firm specific variables. Note that there is no evidence for the frailty in this specification based on the marginal likelihoods.

The coefficients in the frailty model (vi) with firm specific and macro variables are comparable to the ones obtained in the model without frailty as they have similar signs and magnitudes. The loading on the frailty is lower compared the frailty model (iv) which contains only macro variables, while the persistence is slightly higher. In this specification we find evidence of frailty based on the marginal likelihood.

Figure 1 plots the estimated frailty processes from the two models. Although the processes are comparable till the middle of the 1990s, they subsequently show quite different patterns as expected based on the different parameter estimates. The frailty process from the frailty model with macro (iv) hovers around zero. It seems that the frailty process from the model with only macro variables is high during crisis periods to compensate the lagging macro variables. The frailty process for the model with macro and firm specific variables tries to balance the leading firm specific variables by a lower value during the credit crisis of 2008-2009.

To get a further impression about the differences between these models, Figure 2 plots the filtered mean aggregate intensities from the frailty models compared with the filtered mean aggregate intensities from the models with only observables in the intensity specification. The estimated aggregate intensity at time t here means that it is the estimated intensity based on the default experience and observed covariates up until time t. This is different from the way these estimated intensities are usually presented. Usually the aggregate intensity at time t is calculated using the parameter estimates based on the full sample. In general we can say that the savings and loans crisis at the end of the 1980s, the burst of the dotcom bubble around 2000, and the recent financial crisis are picked up by all of the estimated intensity models. However the firm specific variables are clearly useful to explain the dynamics of the

Table 2: The posterior parameter estimates and their 95% credible set for different intensity model specification, estimated on quarterly data from March 1980 till March 2010. The first model (i) is an intensity model without frailty using only macro variables. The second model (ii) is an intensity model without frailty using only macro variables and the median of firm specific variables. The third model (iii) is an intensity model without frailty using only firm specific and macro variables. The fourth model (iv) is a frailty model with macro covariates. The fifth model (v) is a frailty model with macro covariates plus the median of firm specific variables. Finally the sixth model (vi) is a frailty model with macro and firm specific variables. The logarithm of the marginal likelihood is also presented in the last row of the table (Log Evidence).

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Loading on frailty				0.961	0.829	0.707
				(0.465,2.278)	(0.419,1.895)	(0.495,1.016)
Frailty coefficient				0.982	0.970	0.871
				(0.942,0.998)	(0.909,0.998)	(0.719,0.967)
Trasury spread	0.092	-0.012	0.634	-0.01	0.011	0.449
	(-0.031,0.216)	(-0.185,0.162)	(0.482,0.792)	(-0.33,0.298)	(-0.288,0.286)	(0.123,0.749)
Industrial production growth	-1.249	-1.467	-0.502	-1.255	-1.168	-0.813
	(-5.613,2.982)	(-5.693,2.772)	(-4.582,3.761)	(-5.529,2.938)	(-5.389,2.878)	(-4.912,3.38)
1-year S&P 500 returns	-0.324	-0.363	3.562	-0.963	-0.721	4.218
	(-0.934,0.294)	(-1.343,0.618)	(2.894,4.249)	(-1.924,-0.121)	(-2.394,0.92)	(3.034,5.393)
Previous quarter default counts	0.158	0.103	0.026	0.029	0.011	-0.053
	(0.112,0.203)	(0.051,0.156)	(-0.022,0.073)	(-0.047,0.104)	(-0.064,0.085)	(-0.129,0.019)
Median 1-year stock returns		0.525			0.497	
		(-0.442,1.492)			(-0.972,2.033)	
Median distance to default		-0.331			-0.356	
		(-0.492,-0.164)			(-0.693,-0.042)	
Median quick ratio		-0.997			-1.323	
		(-2.883,0.9)			(-4.443,1.764)	
Median log asset value		-0.613			-0.356	
		(-0.944,-0.28)			(-0.931,0.288)	
Median short term debt		0.614			0.591	
		(-3.511,4.852)			(-3.487,4.581)	
1-year stock returns			-2.481			-2.841
			(-2.685,-2.286)			(-3.098,-2.582)
Distance to default			-0.500			-0.503
			(-0.545,-0.455)			(-0.552,-0.452)
Quick ratio			-0.982			-0.991
			(-1.208,-0.757)			(-1.219,-0.762)
Log asset value			-0.472			-0.501
			(-0.546,-0.399)			(-0.579,-0.422)
Short term debt			1.251			1.208
			(0.622,1.864)			(0.581,1.79)
Constant	-4.803	2.601	-2.143	-3.448	1.267	-1.454
	(-5.199,-4.391)	(-0.43,5.754)	(-3.037,-1.26)	(-5.03,-1.532)	(-2.719,5.127)	(-2.682,-0.205)
3-month Treasury Bill	-0.001	-0.12	0.016	-0.172	-0.15	-0.058
	(-0.049,0.048)	(-0.21,-0.034)	(-0.044,0.077)	(-0.383,-0.005)	(-0.332,0.009)	(-0.194,0.064)
Log Evidence	-1800.2	-1778.7	-736.27	-1790.3	-1785.2	-725.83

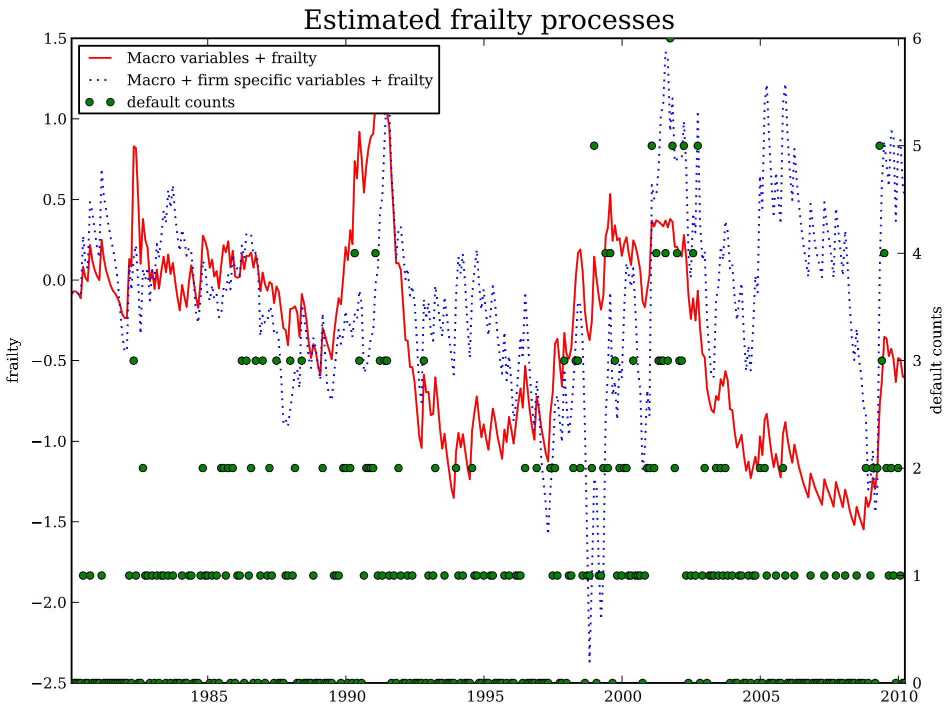

Figure 1: The plot shows the estimated frailty processes from two different models estimated on quarterly data from March 1980 to March 2010 along with the default counts. The first model is a frailty model with macro covariates. The second model is a frailty model with macro and firm specific variables

aggregate intensity process, as the models without the firm characteristics fail to capture the low default intensity in the period after the dotcom bubble and before the financial crisis. Note that, although the firm specific variables are helpful in explaining the default cycle, there is a misalignment in firm specific variables and the actual defaults. The left subplot of Figure 2 shows that the model with firm specific and macro variables overshoots during the Russian financial crisis in 1998 and the financial crisis 2008–2009, and underestimates the default intensity during the burst of the dot com bubble. By adding the frailty factor, these misalignments are corrected.

Figure 3 shows that the sequential posterior mean estimates vary substantially over time. At the beginning of the sample uncertainty about the parameters is huge. Over time the parameters are possibly changing signs and the uncertainty becomes lower.

We conclude we have found evidence that frailty is present in the data even after including the variables suggested by Lando and Nielsen (2010). Figure 4 shows that

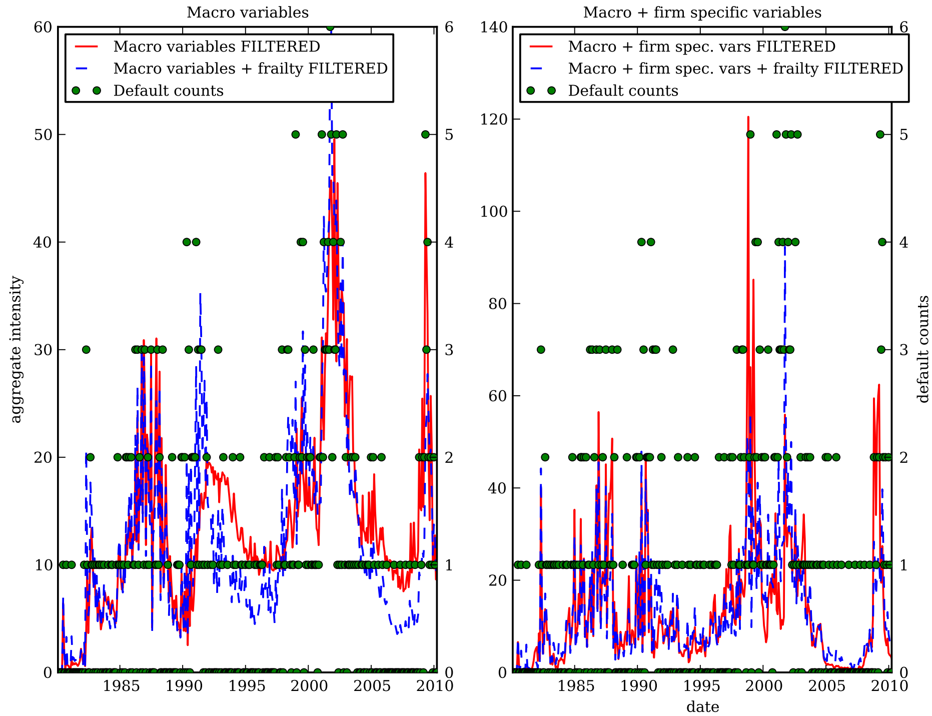

Figure 2: The plot shows the estimated filtered mean aggregate intensity $\sum_{i=1}^{N_t} \lambda_{it}$ from different frailty models compared to estimated intensity from an intensity model without frailty. The models are estimated on quarterly data from March 1980 to March 2010 . The first model is a frailty model with macro covariates. The second model is a frailty model with macro covariates plus the median of firm specific variables. Finally the third model is a frailty model with macro and firm specific variables

the evidence is driven by the periods before the dot com bubble and the recent financial crisis. In these periods the observed defaults are not aligned with the observed macro and firm specific variables and there are serious differences based on the on the non zero frailty values in Figure 1.

6 Explaining the changes of corporate credit spreads

Motivated by the findings of the previous section we test whether frailty can explain some of the variation in changes of corporate credit spreads. We use corporate bond data from TRACE, accounting variables from Compustat and macro variables over the period October 2004 till March 2010. Using the frailty obtained from the previous step we run a pooled panel regression to check if frailty is relevant in explaining corporate credit spreads. We use firms which have both bond and accounting data available. Note that the portfolio of firms which have both bond

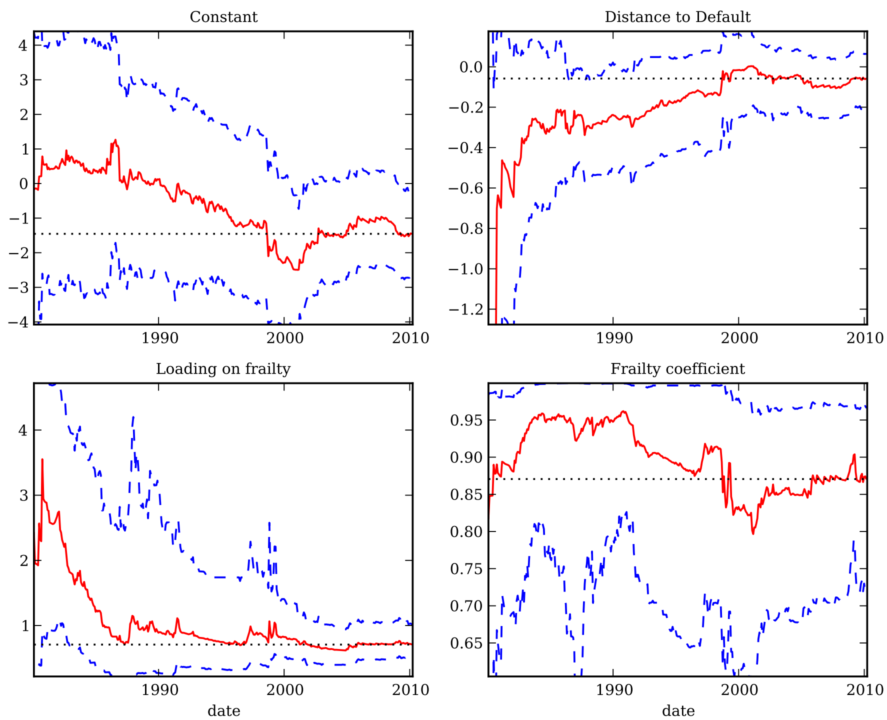

Figure 3: The above plots show the sequential posterior distribution of the constant, the coefficient distance to default, the loading on frailty γ and the frailty coefficient ϕ. The red line shows the posterior mean, the blue dashed line is the 95% credible set while the dotted black line is the level of the posterior mean at the end of the sample

and accounting data is different from the portfolio which we used in the estimation of the intensity model, however we assume these portfolios are the same in terms of their sensitivity to credit risk factors and the estimated frailty process is relevant for both portfolios.

Corporate bonds are sold at a discount because investors require a risk premium for default, recovery and liquidity risk present in these instruments. Assuming that these factors follow Markov processes implies that corporate credit spreads depend on the current level of the factors and on the parameters that govern the dynamics of the risk factors. To explain the changes in corporate credit spreads we use proxies that capture changes in these factors and changes in the parameter estimates.

We use the following firm specific variables. As in the previous section we use 1-year stock returns, changes in distance to default, quick ratio, short term debt, and log asset values as firm specific proxies for credit risk changes. To control for the changes in liquidity we use the illiquidity measure proposed by Dick-Nielsen et al. (2012) This measure is the average of several liquidity measures and it is close to

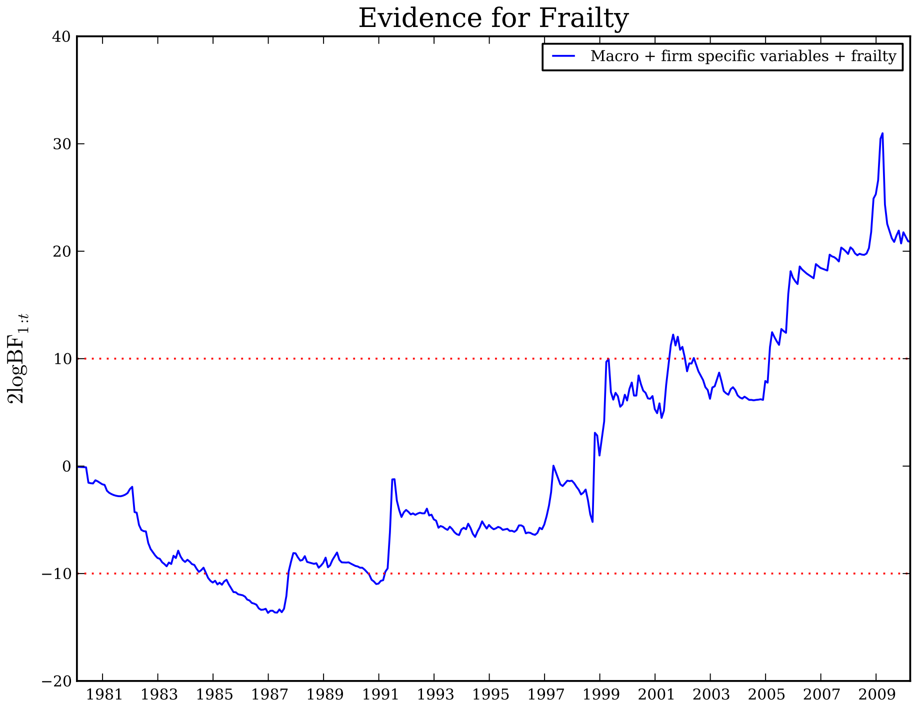

Figure 4: The plot shows two times the logarithm of the Bayes factor for the frailty model with firm specific and macro variables against the intensity model with firm specific and macro variables estimated on monthly data from March 1980 to March 2010. $2\log\mathrm{BF}_{1:t}$ above 10 means strong evidence for frailty.

the first principal component of these measures. Finally we control for changes in 1-year rolling window stock volatility estimates. The inclusion of this variable is motivated by the fact that a corporate bond is equivalent to a portfolio of a risk free bond and a short put option on the assets of the firm with strike price equal to the notional of the bond. As higher volatility implies higher option values, we expect that there is a positive relationship between changes in credit spreads and changes is stock volatility as pointed out by Campbell and Taksler (2003).

We also include several bond specific variables. Bond age is motivated by the fact that older bonds are more likely to be held until maturity. Hence liquidity decreases as the bond gets older and investors have to be compensated for the increased illiquidity. We expect a positive relation between the yield spread and bond age. As an other static liquidity proxy we include in the regression the logarithm of the issued amount. There are several reasons to believe that the investors demand a premium on bonds with smaller issued amounts, see for instance Houweling et al. (2005). First, the issued amount directly effects the trading volume and hence the liquidity of the bond. Second, issued amounts indirectly increase information costs through lowering trading volumes. Finally investors tend to keep bonds with smaller issued amounts in their buy and hold portfolios. We also include time to maturity as a commonly used bond characteristic. We expect positive sign on time to maturity as recently issued bonds are more liquid. We use the coupon rate to control for tax effects. Coupon rates are frequently used as a proxy for tax effects because a tax on the coupon income, requires a higher yield before tax, see (Shiller and Modigliani (1979)). Moreover investors' preference for higher coupon paying bonds can increase demand, hence decrease yields. Based on these considerations we are not certain about the sign of the coefficient on the coupon variable.

We include the following macro variables. For controlling for credit and recovery risk, we include the changes in the macro variables used in the intensity model, namely changes in the 3-month Treasury Bill rate, Treasury spread, and lagged changes in monthly default counts. Moreover, we include industrial production growth and 1-year S&P 500 returns. To control for possible changes in risk appetite, we include the changes int the VIX index. Following Elton et al. (2001) and Avramov et al. (2007) we also include the Fama-French factors and the momentum factor, as credit spreads should contain a compensation for the systemic credit risk exposures. In addition we include the changes in 10 year Treasury bond yields as Longstaff and Schwartz (1995) argue that an increase in the spot rate translates into a higher company value and lower default probability. This suggests a negative relationship between credit spread changes and changes in the 10 year spot rate.

Finally we include the changes in the sequential frailty persistence coefficient ϕ_t and frailty loading estimates γ_t, and the changes in the filtered frailty Δf_t. This is motivated by the fact that assuming a latent frailty variable the credit spread would be a complex function of the current level of frailty and the frailty coefficient and the loading on frailty. This means that the changes in the credit yield spread can be approximated as a linear function of the changes in frailty, the frailty coefficient and the loading on frailty using a Taylor series expansion.

Table 3 summarizes some descriptive statistics of the data. We can categorize the variables into three groups: bond specific variables, bond specific variables which are constant in time and common variables. In general the bond specific variables show more skewness and kurtosis compared to the other two categories. This can be explained by the heterogeneity between the bonds.

Figure 5 shows the filtered frailty and the average yield spread. The picture

Table 3: Descriptive statistics of the firm specific and macro variables used in the panel regression along with the descriptives of the corporate yield spreads. The table show the number of observations (Obs), the mean of the variables (Mean), the standard deviation (Std. Dev), the skewness (Skewness) and finally the kurtosis (Kurtosis).

	Obs	Mean	Std. Dev	Skewness	Kurtosis
Δ Credit spread	14352	-0.051	0.522	-0.158	10.942
1-year stock returns	14352	-7.497	37.763	-0.628	4.736
Δ Distance to default	14352	-0.005	0.408	-0.853	9.360
Δ Quick ratio	14352	0.006	0.092	0.695	16.060
Δ Short term debt	14352	-0.002	0.035	-1.002	16.201
Δ Log asset value	14352	0.004	0.027	1.574	16.166
Δ Illiquidity	14352	-0.003	0.262	0.056	7.642
Δ σ	14352	-0.001	0.033	-0.074	10.290
Bond age	828	3.397	3.639	1.691	2.208
Time to maturity	828	13.321	8.941	0.786	-0.913
Coupon	828	6.290	1.184	0.285	1.865
Log issued ammount	828	6.274	0.615	0.378	0.059
Δ 3-month Treasury Bill	64	-0.027	0.304	-2.181	6.601
Δ 10-year Treasury Bill	64	-0.007	0.281	-0.699	2.797
Δ Treasury spread	64	0.024	0.243	0.902	2.083
Industrial production growth	64	0.000	0.011	1.515	7.156
1-year S&P 500 returns	64	-0.023	0.221	-1.140	0.657
Δ Previous month default counts	64	-0.031	1.817	0.359	-0.040
Δ Monthy default counts	64	-0.016	1.813	0.338	-0.026
Δ VIX	64	0.050	5.056	1.381	6.769
Δ Frailty	64	0.009	0.329	1.572	3.374
Δ Frailty coefficient	64	0.001	0.013	0.891	3.138
Δ Frailty loading	64	0.000	0.006	-0.501	5.181
SMB	64	0.071	0.722	-0.891	7.884
HML	64	0.137	0.597	0.132	3.914
UMD	64	-0.041	0.953	-0.674	1.268

suggests a negative relationship between credit spreads and frailty. This is in line with our previous findings and suggests that frailty is low during crises to compensate the overshooting of firm specific variables, which implies higher model-based default rates compared to the experienced default rates. Hence, the frailty process adjusts the intensity downwards to match the observed default experience.

We estimate several version of the following regression

$$\Delta \text{cs}_{it} = \beta_0 + \beta_1 \Delta f_t + \beta_2 \Delta \phi_t + \beta_3 \Delta \gamma_t + \beta_4 \text{ control varaibles}_{it} + \varepsilon_{it}, \qquad (23)$$

where Δf_t is the change in the frailty, ϕ_t and γ_t are the changes in the sequential frailty persistence coefficient and frailty loading estimates respectively. We use standard errors clustered on months and firms. Table 4 shows the results of the regressions. In the first column we present the results without including frailty and firm specific fixed effects. This model is our benchmark. In this regression the

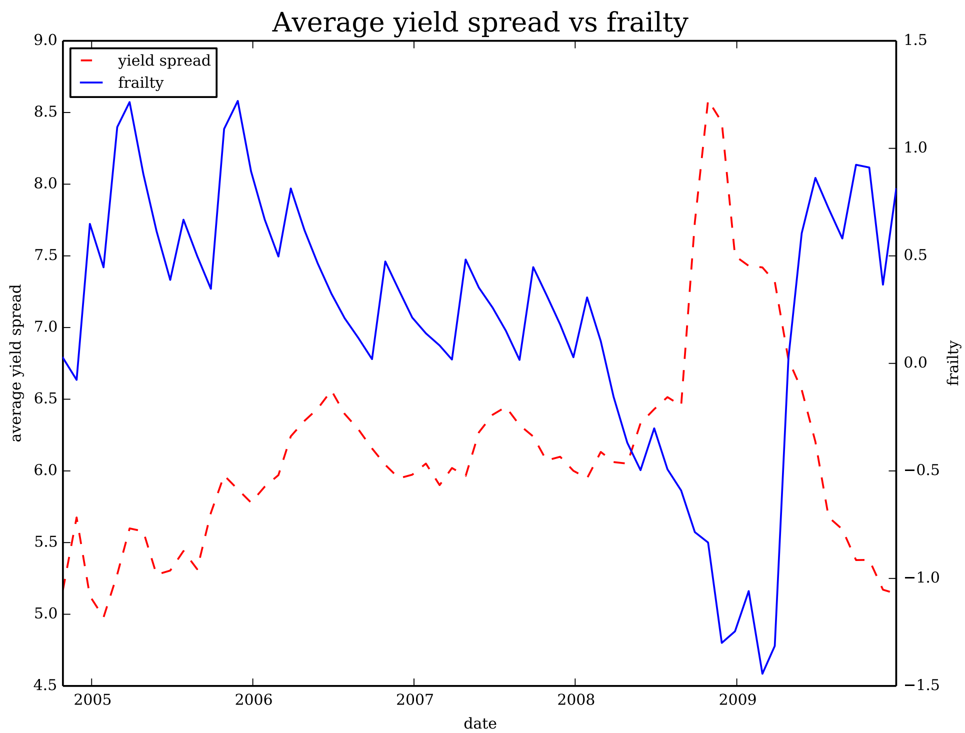

Figure 5: The picture show the average yield spread along with the estimated frailty process from October 2004 to March 2010.

changes in the 3 month Treasury yield, 10 year Treasury yield and the Treasury spread, VIX, log asset value, and the S&P returns are significant at the 1% significance level. The quick ratio is only significant at the 10% significance level. The signs of the significant variables are as expected, except the previous month default counts. The R^2 of this regression is around 0.28.

Column (2) in Table 4 contains the result in case of including frailty in the regression. The estimates on the control variables do not change by much. The sign of the coefficient is consistent with Figures 5 and 2. The right-hand panel Figure 2 revealed that the recursively estimated frailty factor captures the misalignment between default experience and macro plus micro fundamentals. In particular, we see in Figure 5 that the frailty factors picks up on the micro and macro fundamentals overshooting the expected number of defaults, and the frailty correcting for this. As the frailty factor is a stationary process, agents expect it to mean revert and therefore the number of defaults to get more in line with the fundamentals. This expected relative (to fundamentals) increase in defaults correlates directly with the rise in credit spreads at that moment in time, as visualized in Figure 5 and reflected

Table 4: Pooled panel regression of corporate credit spreads (cs) on frailty, credit controls and other control variables. The other control variables include: the liquidity measure, age of the bonds, issued amounts, coupon rate, time to maturity and the Fama French factors (SMB, HML, UMB). A constant is included in the model. Double clustered standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

	(1) Δ cs	(2) Δ cs	(3) Δ cs	(4) Δ cs	(5) Δ cs
Δ Frailty		-0.189**			-0.387***
		(0.091)			(0.119)
Δ Mean reversion			2.451*		15.654***
			(1.472)		(4.028)
Δ Loading				0.324	-23.696***
				(3.200)	(6.995)
Δ 3m Treasury	-0.738***	-0.670***	-0.675***	-0.735***	-0.406**
	(0.194)	(0.195)	(0.197)	(0.202)	(0.185)
Δ Treasury spread	-1.036***	-0.928***	-0.941***	-1.030***	-0.593**
	(0.315)	(0.316)	(0.320)	(0.329)	(0.292)
Industrial production growth	-0.051	-0.055	-0.054	-0.051	-0.064**
	(0.044)	(0.043)	(0.044)	(0.043)	(0.032)
S & P 500	0.005***	0.005**	0.005***	0.005***	0.005***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)
Δ Defaults	0.021	0.024*	0.018	0.020	0.035***
	(0.013)	(0.013)	(0.013)	(0.014)	(0.012)
Previous Δ defaults	0.018	0.026	0.017	0.017	0.044***
	(0.015)	(0.017)	(0.015)	(0.016)	(0.015)
Δ Stock return	-0.001	-0.001	-0.001	-0.001	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Δ Distance to default	0.061	0.056	0.061	0.061	0.062
	(0.054)	(0.054)	(0.054)	(0.054)	(0.054)
Δ Quick ratio	-0.119*	-0.104	-0.127*	-0.120*	-0.104
	(0.067)	(0.066)	(0.068)	(0.069)	(0.064)
Δ Short term debt	-0.021	-0.067	-0.014	-0.021	-0.122
	(0.142)	(0.138)	(0.143)	(0.142)	(0.128)
Δ Log asset	0.585***	0.617***	0.539***	0.581***	0.688***
	(0.192)	(0.197)	(0.185)	(0.193)	(0.192)
Δ σ	1.069	1.168	1.047	1.074	0.775
	(1.060)	(1.015)	(1.062)	(1.055)	(0.965)
Δ 10y Teasury	0.938***	0.970***	0.876***	0.936***	0.753***
	(0.210)	(0.204)	(0.212)	(0.213)	(0.191)
Δ VIX	0.034***	0.032***	0.033***	0.034***	0.033***
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
Other controls	Yes	Yes	Yes	Yes	Yes
Firm fixed effects	No	No	No	No	No
Observations	14,352	14,352	14,352	14,352	14,352
R^2	0.279	0.286	0.281	0.279	0.308

in the negative loading on ΔFrailty in Table 4.

The inclusion of the frailty variable slightly increases the R^2 to 0.286. We have included in the regression the changes in the mean reversion parameter ϕ_t and the changes in the loading on frailty γ_t; see column (3) and (4) in Table 4 respectively. These variables, however, are not significant at the 1% significance level. Finally, we include all of the learning variables jointly in one regression; see column (5). Jointly all of the frailty related variables are significant. The R^2 is about 0.31.

Table 5 presents the results of the regressions with firm fixed effects. Including the firm fixed effects helps to capture firm specific characteristics which are not captured by other control variables. The estimated coefficients are similar to the ones estimated without the fixed effects.

It is possible that the credit spreads of firms with different credit risk react differently to frailty. The credit spread of investment grade firms might not be influenced by frailty in the same way as a speculative grade firm. To take this into account we run a regression with the frailty variables interacted with a dummy that indicates whether the firm is investment grade or not; see column (3) of Table 5

The effect of frailty is negative on both the investment grade and non investment grade bonds. The magnitude of the effect is larger for non investment grade bonds. This also holds for ϕ_t and γ_t. This might be explained by the fact that non investment grade bonds are more sensitive to changes in systemic default risk. However, the effects are not significant for non investment grade bonds using a 1% significance level.

7 Conclusion

In this paper we test for a latent component in corporate default intensities and show that this latent factor explains part of the time series variation in corporate credit spreads. We estimate a dynamic credit risk model using US corporate defaults from January 1980 till March 2010, including frailty as well as firm specific variables used by Lando and Nielsen (2010). We find evidence of a latent frailty process in corporate default intensities. After acknowledging the presence of the unobservable factor we numerically solve the filtering problem of the economic agents and estimate a reduced form asset pricing model that accounts for the fact that agents have to infer the level and the dynamics of the latent factor from observed data. We find that changes in agents' beliefs about the level of the frailty factor are negatively

Table 5: Pooled panel regression of corporate yield spreads (cs) on frailty, credit controls and other control variables.The other control variables include: the liquidity measure, age of the bonds, issued amounts, coupon rate, time to maturity and the Fama French factors (SMB, HML, UMB). A constant is included in the model. Double clustered standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

	(1) Δ cs	(2) Δ cs	(3) Δ cs
Δ Frailty $\times D_{it}^{inv}$			-0.138***
			(0.047)
Δ Frailty $\times(1-D_{it}^{inv})$			-0.252*
			(0.133)
Δ Mean reversion $\times D_{it}^{inv}$			11.026***
			(4.125)
Δ Mean reversion $\times(1-D_{it}^{inv})$			22.296**
			(10.436)
Δ Loading $\times D_{it}^{inv}$			-19.931***
			(7.524)
Δ Loading $\times(1-D_{it}^{inv})$			-23.163
			(20.204)
Δ 3m Treasury	-0.738***	-0.399**	-0.459**
	(0.194)	(0.188)	(0.199)
Δ Treasury spread	-1.036***	-0.587**	-0.676**
	(0.315)	(0.297)	(0.306)
Industrial production growth	-0.051	-0.063*	0.009
	(0.044)	(0.033)	(0.030)
S & P 500	0.005***	0.005***	0.006***
	(0.002)	(0.001)	(0.001)
Δ Defaults	0.021	0.035***	0.020
	(0.013)	(0.012)	(0.014)
Previous Δ defaults	0.018	0.043***	0.015
	(0.015)	(0.015)	(0.009)
Δ Stock return	-0.001	-0.001	-0.000
	(0.001)	(0.001)	(0.000)
Δ Distance to default	0.061	0.062	0.047
	(0.054)	(0.056)	(0.049)
Δ Quick ratio	-0.119*	-0.108*	-0.073
	(0.067)	(0.060)	(0.051)
Δ Short term debt	-0.021	-0.116	-0.151
	(0.142)	(0.134)	(0.120)
Δ Log asset	0.585***	0.677***	0.516***
	(0.192)	(0.236)	(0.186)
Δ σ	1.069	0.760	0.314
	(1.060)	(0.948)	(0.876)
Δ Illiquidity	0.011	0.012	0.009
	(0.025)	(0.024)	(0.024)
Δ 10y Teasury	0.938***	0.751***	0.749***
	(0.210)	(0.194)	(0.208)
Δ VIX	0.034***	0.033***	0.042***
	(0.005)	(0.005)	(0.005)
Other controls	Yes	Yes	Yes
Firm fixed effects	No	Yes	Yes
Observations	14,352	14,352	12,995
R^2	0.279	0.320	0.342

correlated with corporate credit spread changes.

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