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Measuring flight-to-quality with Granger-causality tail risk networks*

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Abstract

We introduce an econometric method to detect and analyze events of flight-to-quality by financial institutions. Specifically, using the recently proposed test for the detection of Granger causality in risk (Hong et al. 2009), we construct a bipartite network of systemically important banks and sovereign bonds, where the presence of a link between two nodes indicates the existence of a tail causal relation. This means that tail events in the equity variation of a bank helps in forecasting a tail event in the price variation of a bond. Inspired by a simple theoretical model of flight-to-quality, we interpret links of the bipartite networks as distressed trading of banks directed toward the sovereign debt market and we use them for defining indicators of flight-to-quality episodes. Based on the quality of the involved bonds, we distinguish different patterns of flight-to-quality in the 2006-2014 period. In particular, we document that, during the recent Eurozone crisis, banks with a considerable systemic importance have significantly impacted the sovereign debt market chasing the top-quality government bonds. Finally, an out of sample analysis shows that connectedness and centrality network metrics have a significant cross-sectional forecasting power of bond quality measures.

JEL codes: G00; G01; G21; H63; C12;

Keywords: flight-to-quality; sovereign debt crisis; systemic risk; Granger causality; bi-partite networks; illiquidity; fire sales;

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1 Introduction

The surge of market distress and the rapid spread of uncertainty may lead a significant number of market players to rethink their priorities and investment strategies triggering a rebalancing of their portfolio toward safer type of assets (Caballero and Krishnamurthy, 2008). When investors have similar portfolios, coordinated rebalancing might lead to massive sales and purchases of assets (Cont and Wagalath, 2013, 2014). In the presence of finite liquidity, these “fire sale” episodes can lead to sudden and large price movements, with a consequent significant increase of financial systemic risk. In particular, in periods of financial turmoil, financial investors try to liquidate risky assets and to purchase safer and less risky ones. Episodes of this type are commonly referred to as “flight-to-quality” and can play a prominent role in the propagation and deepening of financial crises. Thus, the ability of identifying and possibly anticipating (at least partially) such phenomena is of great importance also in the context of early-warning and monitoring of systemic risk (Demirgüç-Kunt and Detragiache, 1998; Kaminsky and Reinhart, 1999; Harrington, 2009; Scheffer et al., 2009; Barrell et al., 2010; Duttagupta and Cashin, 2011; Kritzman et al., 2011; Allen et al., 2012; Arnold et al., 2012; Bisias et al., 2012; Scheffer et al., 2012; Merton et al., 2013; Oet et al., 2013). Moreover the identification of flight-to-quality episodes is of great importance as a measure of the market perception of the quality of the assets.

In empirical studies of systemic risk there are two possible approaches. The first one is characterized by the adoption of proprietary data on portfolio compositions, exposures, trades, etc., data typically unavailable or partially available only in central banks. The second approach makes use of econometric analyses of publicly available market data (e.g. prices and/or volumes) in order to identify, characterize, and possibly forecast episodes of systemic risk. In this paper we follow the second approach to identify flight-to-quality episodes, focusing our attention to the relation between banks and sovereign bonds in the period 2006-2014.

Our analysis starts from the simple consideration that, prior and during a crisis event, systemically relevant banks are likely to experience large drops in their equity values. Accordingly, crises are identified by events originated in the left-tail of the equity log-returns distribution of global systemically important banks¹. Since big market players are usually levered institutions pursuing a target or pro-cyclical leverage² strategy (Adrian and Shin, 2010, 2014; Greenwood et al., 2015) and since sovereign debt securities form a considerable fraction of banks’ total assets, a portfolio rebalancing of a bank will most likely involve the trading of large quantities of sovereign bonds. Due to market impact and assets’ illiquidity, large movements of bond prices (and hence of bond yields) are expected as a consequence of the flight-to-quality behavior of banks triggered by the large equity drop. Risk exposure can, in fact, be reduced by selling risky assets and moving capitals toward safer investments. Hence, large sovereign bond yield variations tend to occur *after* periods of financial distress in which many large banks are hit by dramatic losses in their equity capitals.

As a motivation for the proposed econometric method, first we present a stylized model of flight-to-quality inspired by the theoretical models of Corsi et al. (2013) and Lillo and Pirino (2015) (which are, in turn, inspired by previous works by Adrian and Shin, 2010, 2014; Greenwood et al., 2015, among others) of the feedback between equity of institutions and price of assets. Specifically we show, by means of a simple portfolio optimization model, that if two types of assets are available in the economy, a safe one and a risky one, a VaR-constrained profit-maximizer will react to a drop of the equity by purchasing the safe asset and selling the risky one. In fact, the model predicts that equity drops of financial institutions will simultaneously increase the demand of safe assets and the supply of riskier assets. Because of market impact, this in turn drives an upward (downward) movement of price of safe (risky) asset. In order to econometrically identify

¹In particular, we rely on the definition of G-SIBs defined by Basel III regulatory framework. For more detail, see <http://www.bis.org/bcbs/base13.htm>.

²In periods of extraordinary distress and uncertainty, de-leveraging is a common strategy also for financial institution not actively managing their leverage, see (Chen et al., 2014).

this chain of events from market prices, we propose to use Granger-test for tail dependence of Hong et al. (2009). Applied to the banks-sovereign bonds relation, the test identifies when a tail event in the distribution of the equity log-return of a bank helps forecasting a tail event in the distribution of the yield variations of a given sovereign bond.

Considering a set of banks and sovereign bonds in a systemic perspective, the information on flight-to-quality episodes can be described by a bank-asset bipartite network³ where the link between a bank and a bond indicates the existence of Granger causality in tail. More precisely, we define connectivity and centrality measures of the Granger-causality networks that have a simple economic interpretation in terms of indicators of distressed selling and distressed buying. In addition, we adopt country ratings from Standard & Poor's (S&P) to discriminate between "good" and "bad" sovereign debts. We can thus identify periods when a significant amount of distress selling has hit the class of "bad" bonds while a significant amount of distress buying was simultaneously hitting the "good" ones. This double effect corresponds to a natural characterization of flight-to-quality, where the general distrust pushes investors toward low-risk assets. Our proposed flight-to-quality measure possesses several appealing features: (i) it is computed from standard equity and bond (daily) market prices; (ii) it is able to identify different patterns of flight-to-quality behavior; (iii) allows the derivation of several connectedness and centrality network metrics having significant cross-sectional forecasting power for the bond quality.

Our paper has been particularly inspired by two important works belonging to two different streams of the literature which we attempt to connect: the paper of Beber et al. (2009) on flight-to-quality and the paper of Billio et al. (2012) on networks of Granger-causality.

The analysis by Beber et al. (2009) sheds insightful light on how liquidity and quality are chased in the market. Unusual capital flows are used in Beber et al. (2009) to proxy flights, both to liquidity and to quality. They document that investors care of both liquidity and quality, but they do it at different times. More precisely, in period of market distress characterized by a high level of uncertainty, investors require liquidity rather than quality. Besides, liquidity plays a non-trivial role even in the pricing of bonds during period of increased uncertainty. Defining an econometric measures of flight-to-quality based on easily available daily market prices, considerably reduces the data requirements in our approach since the detailed information on the order flow is now inferred from data. This allow us to considerably extend the time span of our analysis compared to the one in Beber et al. (2009) considering the equity and bond market developments from the beginning of 2006 to February 2014. As a consequence, our analysis profits from the presence of two important periods of financial distress. Complementing the results of Beber et al. (2009) we document that investors (in our case systemically important banks), do chase for quality during financial crisis, but they do it in different ways: in some cases, such as during and after the Lehman crisis, they simply chase quality by buying sovereign debt of any type, while in others, such as during the European sovereign debt crisis, they require only top-quality sovereign debt, i.e. the flight-to-quality occurs exclusively toward AAA-rated sovereign bonds.

Networks of Granger-causality have been introduced in systemic risk studies by Billio et al. (2012) to identify and quantify periods of financial turbulence, characterized by abnormal levels of Granger inter-connectedness among equities of hedge funds, banks, broker/dealers, and insurance companies. Our paper, although being inspired by the analysis of Billio et al. (2012), moves away from it in at least two respects: first, we adopt a bipartite network of equities and bonds, a choice dictated by the will of investigating the effect of crises on sovereign debt and, second, we adopt the Granger-causality test in the tail by Hong et al. (2009), since we believe that it is suited to describe events pertaining to a crisis, hence of extraordinary nature. Moreover, the test is able to identify the "sign" of the causality, in the sense that it can distinguish

³A bipartite network is a graph whose vertices can be divided into two disjoint sets such that every edge connects a vertex in one set to one in the other set.

if an event in the right or in the left tail of the caused variable is anticipated by an event in one of the two tails of the causing variable.

We adopt four additional definitions of bond quality based on either the correlation between CDS spreads and the bond yields or the bond yield volatility and the yield spreads, defined as the difference between a government bond yield and the German bond yield at the same maturity. This gives us the possibility of having a time-varying measure of quality independent from the S&P rating of the country and, hence, to test the out-of-sample forecast performances of the newly defined network centrality measures. We find that during period of crises, both for the 2007 but especially for the Eurozone crisis, the Granger centrality measures show a significant forecasting power on the bond quality measures.

It is important to stress that the method we present is more general than the specific application to flight-to-quality episodes of systemically important banks toward sovereign bonds and can be applied to other financial institutions (e.g. mutual funds, hedge funds, etc.) and other asset classes (e.g. equities, corporate bonds, etc.). In particular it could be used to identify episodes of fire sale in the financial market, not necessarily related to flight to quality.

It is however worth remembering that the econometric evidence of a tail causality does not necessarily implies that the bank has executed a distressed selling (or buying) on the corresponding bond. We do not observe directly trading activity and thus our approach should be interpreted with all the limitations of the original causality by Granger (1969). In other words, our method must be interpreted with an “as if” approach. Nevertheless, the presence of tail causality between the time series represented by two nodes is an important indicator of the propagation of risk between them.

The paper is organized as follows: Section 2 is dedicated to outline the motivational portfolio optimization model. The main implication of the model is summarized in Proposition 1, whose proof is given in Appendix A. In Section 3 we present the Granger-causality in tail as our method to identify econometrically portfolio rebalancing and in Section 4 we present the investigated dataset. In Section 5 we describe the structure of the bipartite network of tail causality and in Section 6 we present our definitions of flight-to-quality metrics and we study their dynamics. Section 7 describes the out of sample forecasting tests for the quality of the bond based on the properties of the bipartite network. Finally, in Section 8 we draw some conclusions.

2 A simple model of flight-to-quality

As anticipated in the Introduction, the first contribution of the present paper is a portfolio optimization model of a simplified economy in which an investor can choose between two types of assets: a safe one with low expected return and a risky one with high expected return. In line with the recent theoretical and empirical literature on bank behavior (Danielsson et al., 2004; Brunnermeier and Pedersen, 2009; Adrian and Shin, 2010; Duarte and Eisenbach, 2013; Adrian and Shin, 2014), financial institutions are confronted with a Value at Risk (VaR) type of constraints. The final purpose is to understand the consequences of such a VaR constraint on the decision making process of a profit maximizer.

Consider then an investor who can invest only into two assets a and b . We proxy riskiness of the investments by their volatilities. Hence let σ_a and σ_b be the volatilities of, respectively, asset a and b , and assume (without loss of generality) that $\sigma_a < \sigma_b$. Assuming positive risk premia and denoting by μ_a (resp. μ_b) the expected return of asset a (resp. b) we have that $\mu_a < \mu_b$. Let ω be the percentage of total asset invested in the safe asset a and $1 - \omega$ the corresponding percentage invested in the risky asset b . The investor has to find the optimal total asset A and the optimal ω that solve

$$\begin{aligned} \max_{A, \omega} \quad & A \boldsymbol{\mu}' \boldsymbol{\omega} \\ \text{s.t.} \quad & \alpha A \sqrt{\boldsymbol{\omega}' \boldsymbol{\Sigma} \boldsymbol{\omega}} \leq E, \end{aligned} \tag{1}$$

where E is the total equity provided to the investor, α is a parameter that determines which quantile⁴ of the VaR is used in the constraint, the matrix $\boldsymbol{\Sigma}$ is the variance-covariance matrix of the two assets

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{pmatrix},$$

the two vectors $\boldsymbol{\mu}$ and $\boldsymbol{\omega}$ are, respectively, the vector of asset returns and the vector of portfolio weights

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_a \\ \mu_b \end{pmatrix}, \quad \boldsymbol{\omega} = \begin{pmatrix} \omega \\ 1 - \omega \end{pmatrix},$$

and, finally, the notation $\boldsymbol{\omega}'$ indicates the transpose of $\boldsymbol{\omega}$. Note that we are not imposing $\omega > 0$, since a short-position may be profitable for the investor. Under this general setting we are able to prove the following

Proposition 1 *A profit-maximizer that allocates the available resources according to (1) always reacts to equity drops with a flight-to-quality. In formula*

$$\frac{d\omega}{dE} < 0. \tag{2}$$

Proof. See Appendix A.

The implications of the model are summarized by inequality (2). During periods of financial distress ($dE < 0$) the VaR-constrained investor will simultaneously purchase safe investments ($d\omega > 0$) and sell the risky ones ($d(1 - \omega) < 0$). In the context of the bi-partite network of systemically relevant banks and sovereign bonds analyzed in this paper, we expect that, during financial crises, a simultaneous distressed selling of risky bonds is accompanied with a massive purchase of good quality bonds. We stress that, in the model presented above, the investor is allowed to choose the optimal A . Hence, as a compensation of the equity drop, the investor could in principle choose to solely reduce the optimal total asset A , leaving the portfolio allocation $\boldsymbol{\omega}$ untouched. In practice, the total asset is reduced by the same amount of the equity drop⁵ together with a reduction of the exposure to the risky bond. Simply put, the flight-to-quality phenomenon is not affected by the possibility of choosing the optimal A .

The model helps understanding that an equity drop of a financial institution will lead to a liquidation of risky assets and purchases of safe assets. Because of finite liquidity, this excess demand/supply will move upward (downward) the price of safe (risky) assets. It is therefore natural to search, in market data, for dependencies between extreme movements of the equity of financial institutions and extreme movements of asset prices, as an indication of flight-to-quality episodes. This is the focus and the main goal of the next Section.

⁴By fine-tuning of the constant α a supervisory authority may decide to relax or to tighten the VaR constraint and, as a consequence, to reduce or to increase the maximum market risk of the bank.

⁵This is immediate since, in obtaining inequality (2), we are assuming an almost constant debt (see Adrian and Shin, 2010).

3 Tail Granger-causality for identifying distressed portfolio rebalancing

In this paper, in order to identify distressed portfolio rebalancing, we adopt a purely econometric approach that only requires standard time series of banks' equities and sovereign bond yields. This choice has the advantage of being implementable at frequencies from moderate to high (weekly, daily or even more, depending on the availability of data) without requiring any not publicly available dataset, such as order flow data, which are notoriously very hard to obtain. As anticipated in the introduction, we look for those chains of events in which a large negative equity drop of a systemically important bank causes a significant variation (positive or negative) of a given sovereign bond yield. We adopt the statistical test proposed by Hong et al. (2009), which is summarized below. The final outcome of the econometric machinery is a bi-partite network of Granger causalities in tail (or risk). We interpret the presence of a link in such network as an indirect evidence of a distress buying or selling of the bond by the bank with the interpretative precaution mentioned in the Introduction.

The Granger-causality test for tail dependence of Hong et al. (2009) works as follow. Given two time series $\{Y_{1,t}\}_{t=1}^T$ and $\{Y_{2,t}\}_{t=1}^T$ (think at the former as the series of bond yield variations and at the latter as the series of equity log-returns), the first step of the procedure is to identify which are the extreme events in the history of both. For this purpose, following the approach of Hong et al. (2009), we estimate a parametric model for the Value-At-Risk of $\{Y_{1,t}\}_{t=1}^T$ and $\{Y_{2,t}\}_{t=1}^T$. For the parametric model, as well as the numerical routines, we borrow from the popular approach developed by Engle and Manganelli (2004). For more detail about the VaR model and the corresponding parameter estimates we refer to Appendix B. The outcome of the Engle and Manganelli (2004) procedure is a parametric estimate of the conditional VaR series $\left\{V_{i,t}^{(\alpha)}\right\}_{t=1}^T$ defined in the standard fashion

$$\text{Prob}\left[Y_{i,t} \leq -V_{i,t}^{(\alpha)} \mid \Omega_{t-1}\right] = \alpha, \quad i = 1, 2,$$

where Ω_t denotes the information available at time t and α defines the tail of the distribution. We choose $\alpha = 10\%$ as a trade-off between the necessity of focusing on extreme events and that of having a sufficiently high number of observations. Once the two conditional VaR series have been estimated we can apply the one-way Granger-causality test by Hong et al. (2009). The test compares the null

$$\mathbb{H}_1^0 : \text{Prob}\left[Y_{1,t} < -V_{1,t}^{(\alpha)} \mid \{Y_{1,t-k}\}_{k=1}^{t-1}\right] = \text{Prob}\left[Y_{1,t} < -V_{1,t}^{(\alpha)} \mid \{Y_{1,t-k}, Y_{2,t-k}\}_{k=1}^{t-1}\right] \quad (3)$$

against the alternative

$$\mathbb{H}_1^A : \text{Prob}\left[Y_{1,t} < -V_{1,t}^{(\alpha)} \mid \{Y_{1,t-k}\}_{k=1}^{t-1}\right] \neq \text{Prob}\left[Y_{1,t} < -V_{1,t}^{(\alpha)} \mid \{Y_{1,t-k}, Y_{2,t-k}\}_{k=1}^{t-1}\right]. \quad (4)$$

The interpretation of \mathbb{H}_1^0 and \mathbb{H}_1^A is straightforward. If the null hypothesis is rejected with some confidence, in the time interval $[t_{\text{start}}, t_{\text{end}}]$, this means that the information of the occurrence of a large event in $Y_{2,t}$ during $[t_{\text{start}}, t_{\text{end}}]$ has significantly impacted the probability of a future occurrence of an extraordinary event in $Y_{1,t}$. Note that the conditioning information includes at most instant $t - 1$, while the conditioned event $\left\{Y_{1,t} < -V_{1,t}^{(\alpha)}\right\}$ is occurring at t , whence the genuine causality of the approach. Notice also that in the test we are not considering simultaneous dependencies.

Define the hit series as $Z_{i,t} \equiv \mathbb{1}_{\{Y_{i,t} < -V_{i,t}^{(\alpha)}\}}$ for $i = 1, 2$ and $t = 1, \dots, T$, and let $\hat{\rho}(j)$ be the sample cross-correlation function between $\{Z_{1,t}\}_{t=1}^T$ and $\{Z_{2,t}\}_{t=1}^T$ at positive lag j , i.e. consider

$$\widehat{C}(j) = T^{-1} \sum_{t=1+j}^T (Z_{1,t} - \alpha_1) (Z_{2,t-j} - \alpha_2), \text{ with } j = 1, 2, \dots, T-1,$$

where $\alpha_i = T^{-1} \sum_{t=1}^T Z_{i,t}$. The sample cross-correlation function is thus written as

$$\widehat{\rho}(j) = \frac{\widehat{C}(j)}{S_1 S_2},$$

where $S_i^2 = \alpha_i (1 - \alpha_i)$. Hong et al. (2009) show that, under \mathbb{H}_1^0

$$Q_1(M) = \frac{T \sum_{j=1}^{T-1} k\left(\frac{j}{M}\right)^2 \widehat{\rho}(j)^2 - C_{1,T}(M)}{D_{1,T}(M)^{\frac{1}{2}}} \xrightarrow{D} N(0, 1)$$

when both the number of observation $T \rightarrow \infty$ and the bandwidth $M = cT^\nu \rightarrow \infty$ ($c > 0$, $0 < \nu < \frac{1}{2}$), where $k(x)$ a suitable kernel function⁶, and $C_{1,T}(M)$ and $D_{1,T}$ are known constants. Notice that in the previous equation we consider only positive values of the lag j , i.e. lagged correlation between past observations of $Z_{2,t}$ and future observations of $Z_{1,t}$ (since we are testing the causal relation of the second variable on the first one).

Under \mathbb{H}_1^A it is

$$\frac{M^{\frac{1}{2}}}{T} Q_1(M) \xrightarrow{P} \frac{1}{\left(2 \int_0^\infty k(z)^4 dz\right)^{\frac{1}{2}}} \sum_{j=1}^{\infty} \widehat{\rho}(j)^2, \quad (5)$$

which implies that the test has asymptotic unit power at any confidence level. If $Q_1(M) > c_\beta$, where c_β is the β -quantile of a Normal distribution function with zero mean and unit standard deviation, then we say that $\{Y_{2,t}\}_{t=1}^T$ Granger-causes the series $\{Y_{1,t}\}_{t=1}^T$ in the tail (or in risk). Throughout the analysis we choose $\beta = 95\%$ and hence $c_\beta = 1.6449$. Moreover, following the small-sample properties of the test as reported by Hong et al. (2009), we set $M = 5$ for the value of the bandwidth parameter.

Since our interest is in exploring causal relationships between equities and bonds, we have to specify both the direction of causality and which tail (left or right) of the two distributions are involved in the test. In this paper, we will limit our analysis to causality links between the left tail (losses) of the bank equity distribution and both the left and right tails of bond yield distribution. The economic interpretation is straightforward. In the former case, a negative shock in equity capital for institution j is causing large negative bond yield variations for country i , and this is interpreted as if the bond is being bought. Vice versa, in the latter, a negative shock in equity capital is causing a sudden increase of bond yields, and this is interpreted as a distressed selling against the corresponding sovereign debt. For the sake of readability, we will denote a distressed buy of bank j of bonds i as $(E_j \implies B_i^{\text{Buy}})$ while a distress selling bank j of bonds i as $(E_j \implies B_i^{\text{Sell}})$.

For completeness and in order to compare our results with those of Billio et al. (2012), we also perform standard Granger (1969) causality tests, whose theory is briefly summarized in Appendix D. In this case, if the j -th equity log-returns series Granger-causes the i -th series of sovereign bond yield variations, we write $(E_j \implies B_i)$. Contrary to Hong et al. (2009), the standard Granger (1969) causality tests only controls for causality between the central moments of the pair of distributions under analysis.

Since we aim at implementing a dynamical analysis, we paid particular care in the construction of both the VaR measure and the Granger-causality test to fully preserves causality using, at any point in time, only past information. Appendix B describes in detail the solution adopted. It is important to mention that we circumvent contingent problems generated by the asynchronicity of the data by sampling all time series

⁶In our analysis we adopt the Daniell kernel $k(x) = \sin(\pi x)/x$.

every two-days, hence bond yield variations and equity log-returns must be thought as variations on a time scale of two days.

4 The Dataset

Our dataset is composed of $j = 1, \dots, 33$ equities of global systemically important banks (G-SIBs) defined by Basel III regulatory framework and $i = 1, \dots, 36$ sovereign debt bonds (with a maturity of 5 years) of different countries in America, Europe and Asia. We indicate with $\gamma_{i,t}$ the nominal 5-years maturity bond yield of country i at day t . For each country i we have also at our disposal the daily time series of five-years maturity CDS spread, a quantity that we indicate as $\mathcal{S}_{i,t}$ henceforth. Detailed informations, along with Tables with summary statistics, on the three blocks of data are provided, for the sake of readability, in Appendix C.

Finally, we mention that in Section 7.1, in order to properly define the quality measures based on the correlation between CDS spreads and bond yields, we require a proxy for the risk-free interest rate r . For each bond, depending on the currency, we use the zero-coupon curve of the corresponding maturity and currency derived by a bootstrapping procedure⁷.

5 Bipartite networks of tail causality

In order to build a network of Granger-causality in the tail, we construct a sequence of time-windows of three years length⁸. For a given time-window t , we compute the networks of the significant causal links in the tail $(E_j \implies B_i^{\text{Sell}})_t$ and $(E_j \implies B_i^{\text{Buy}})_t$, where now the additional subscript t is indicating that the causal networks are time-dependent since they are computed using observations prior to the end of the t -th time-window⁹. In addition, we also compute the network from the standard Granger (1969) causality test $(E_j \implies B_i)_t$.

Six snapshots of the equity-bond bipartite network of tail risks are shown in Figure 1 (for the sake of exposition the networks based on the Granger (1969) test are omitted). Left and right columns of Figure 1 refer to the two types of tail causal relationships investigated. More precisely, a link in the left column is a $(E_j \implies B_i^{\text{Buy}})_t$ causal link, while in the right column it is a $(E_j \implies B_i^{\text{Sell}})_t$ causality. When hit by a causal link, bonds are colored according to the S&P rating attached to the bond in the corresponding time-window (see caption for more details). Finally, we indicate the degree of each node of the bipartite graph in brackets (when different from zero).

The three periods displayed are selected with a simple criterion. The first one (first row) refers to the period 2003-2006 and thus it is not associated with any financial crisis. Interestingly the network shows a relatively small number of active links. The other two refer to 2006-2009 (middle row) and 2009-2012 (bottom row) and thus are associated with two phases of the financial distress, namely the events around the Lehman default and the Eurozone sovereign crisis. In these cases the number of causal links is significantly higher. The purpose is to give a visual perception of the differences in the topology of the network between normal and crisis periods. Specifically, during episodes of financial turmoils, it is evident that $(E_j \implies B_i^{\text{Buy}})_t$ causal links are preferentially directed toward AAA bonds, while $(E_j \implies B_i^{\text{Sell}})_t$ causal links are mainly oriented toward bonds with rating ranging from AA to BB. This evidence anticipates the results of Section 6, where

⁷All the zero-coupon curves are elaborated, via a bootstrapping procedure representing the best-practice in the industry, by a dedicated desk of UniCredit Group.

⁸The choice of three years balances the trade-off between the power of the Granger test (as reported in the simulation study of Hong et al., 2009) and the need of localizing periods of financial distress.

⁹Since each time series (either on the equity and on the bond side, see Tables 4 and 5) starts at a different day, for any equity-bond pair we check if the overlap of the two time series contains more than 10 observations, which in the two-days subsampled grid roughly corresponds to one-month. However, in the great majority of cases, we get almost a full overlap which corresponds to 390 observations (three years of 2-working-day sub-sampled observations).

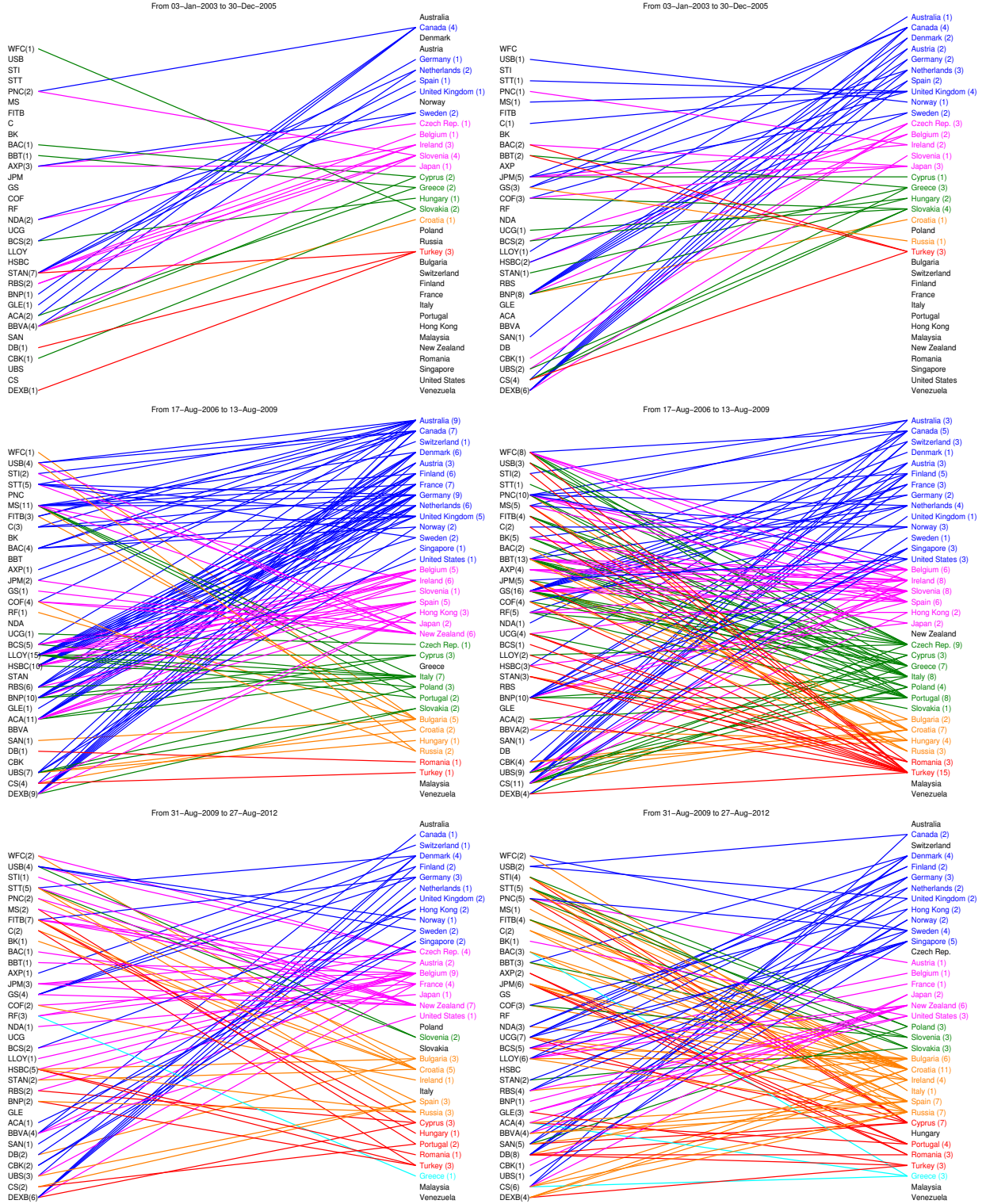


Figure 1: Six snapshots of the causal network described in Section 5. The left column reports networks based on the $(E_j \implies B_i^{\text{Buy}})_t$ causal relationship, while the right column refers to the $(E_j \implies B_i^{\text{Sell}})_t$ one. Hence a link in the left (resp. right) column is a proxy for a distressed buying (resp. selling). For each snapshot we report in the title the period used to compute the network. For comparison, each row in the plot corresponds to a given period. The first row displays a period that is not associated to a strong financial crisis. The other two are chosen among (non-overlapping) periods of financial distress. When hit by a link, sovereign bonds (and the corresponding link) are coloured according to the attached S&P rating (blue = AAA, magenta = AA, green = A, orange = BBB, red = BB) while, in the other case, they are left black. We report node degrees for both banks and bonds in brackets (only when different from zero).

we propose a formal test to confirm that, during the depicted periods, the number of distress buying directed toward AAA rated bonds and that of distress selling toward the remaining ones is not compatible with a random graph.

As in Billio et al. (2012), we are interested in measuring of the density of the network (also called connectedness) whose deviation from the expected value under the null of no causal relationship may signal the presence of systemic events or distress. As a first investigation we define the two quantities

$$\begin{aligned} D_t^{\text{Sell}} &\equiv \frac{1}{N_t^E N_t^B} \sum_{j=1}^{N_t^E} \sum_{i=1}^{N_t^B} (E_j \implies B_i^{\text{Sell}})_t, \\ D_t^{\text{Buy}} &\equiv \frac{1}{N_t^E N_t^B} \sum_{j=1}^{N_t^E} \sum_{i=1}^{N_t^B} (E_j \implies B_i^{\text{Buy}})_t, \end{aligned} \quad (6)$$

where N_t^E and N_t^B represent, respectively, the total number of available equities and bond series in the t -th network. The two quantities defined in (5) are the percentage of validated causal links according to the either $(E_j \implies B_i^{\text{Sell}})_t$ or $(E_j \implies B_i^{\text{Buy}})_t$. For completeness we also define the percentage of validated links according to the original Granger (1969) definition

$$D_t = \frac{1}{N_t^E N_t^B} \sum_{j=1}^{N_t^E} \sum_{i=1}^{N_t^B} (E_j \implies B_i)_t. \quad (7)$$

Figure 2 reports the two measures of connectedness defined in (5) and the Granger (1969)-based measure defined in (7), as a function of time. Note that, being all statistical tests for each equity-bond pair performed with a confidence level of 5%, under the null of no-causality we expect to find, for the three connectedness measures, a value around 5%. In fact, the confidence levels under the null of no-causality may be higher due to finite-sample effects or estimation errors. For this reason we compute confidence levels of the three types of causality using a bootstrap procedure described in Appendix E. As a results we obtain three different levels, one for each causality, reported in Figure 2 with the same line styles of the corresponding measure of connectedness. The levels for the two measures defined in (5) are slightly greater than that of the Granger (1969)-based measure mainly because of the error involved in the estimation of the VaR model but also because the tail-test is built from a smaller statistics.

Figure 2 witnesses an interesting empirical evidence. If we interpret D_t^{Sell} and D_t^{Buy} as, respectively, indicators of periods of general distress selling and distress buying, the former peaks during the Eurozone crisis while the latter during the 2007 – 2008 financial market crisis. This evidence provides a first indication of the fact that the two crisis periods are associated with different types of flight-to-quality behaviors: the first is characterized by a strong increase in sovereign bond purchases which is not accompanied by a substantial rise in sovereign bond selling; while the second crisis features the contemporaneous presence of both generalized distress buying and distress selling of sovereign bond. We will further investigate and discuss this evidence in the next section. Finally, we note that the original Granger (1969)-based centrality measure produces a weaker signal compared to the tail Granger-causality measures, being significant only during the Eurozone crisis, and appear to be highly related with the indicator of distressed selling.

6 Flight to quality

In this section we develop our econometric measure of flight-to-quality built upon the Granger causal networks previously defined. A flight-to-quality episode is commonly referred to as a conveyance of capital from risky

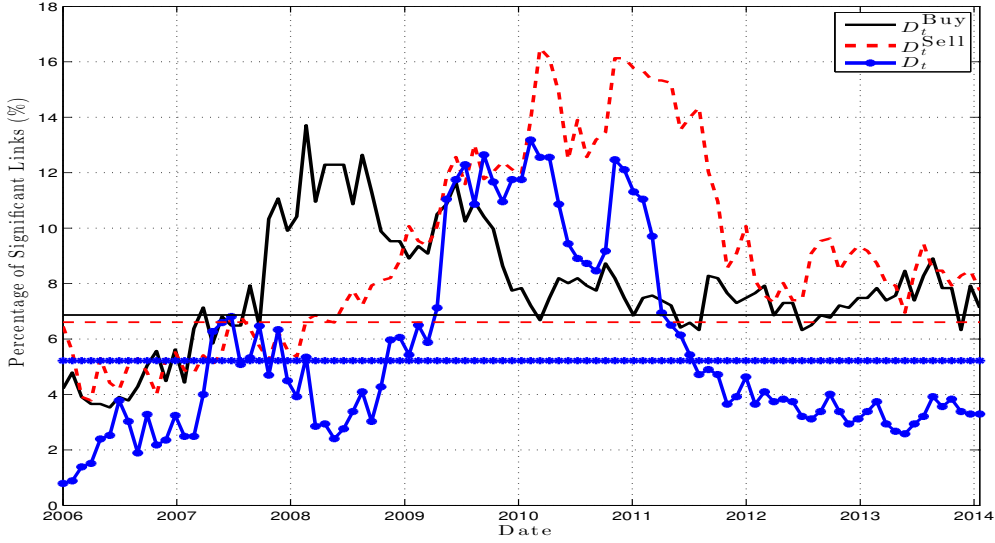


Figure 2: The black continuous and the red dotted lines represent, respectively, the percentage D_t^{Buy} of 95%-significant ($E_j \implies B_i^{\text{Sell}}\big)_t$ and the percentage D_t^{Sell} of ($E_j \implies B_i^{\text{Buy}}\big)_t$ 95%-significant causal links defined in equation (5). The blue line with filled circles plots D_t , that is the percentage of validated links according to the Granger (1969)-based causality defined in equation (7). The dates reported in the horizontal axis correspond to the end of the time-window. Hence, for example, the level of the black line around mid-2008, which is approximately 12%, is computed using observation from mid-2005 to mid-2008. Horizontal lines report the bootstrapped 5% confidence level of each measures (see Appendix E).

assets to more secure ones thus accepting to receive lower expected returns. Such events are triggered by particularly distressed states of the market in which the risk aversion of investors may suddenly increase. As clearly spelled out in Anderson and Liu (2013), [...] *in times of turmoil, investors accept zero or negative nominal yields as a fee for safety.*

The crucial point here is to understand which kind of quality is sought by market players in each period. First we need to identify a proxy for the quality of sovereign bond. For this analysis we use the historical S&P's ratings. More precisely, for each bond in the dataset, we have at our disposal the historical S&P's ratings divided into 10 different rating classes¹⁰. We aggregate these 10 rating classes into two broad categories which we generically denote as “good” and “bad” bonds. Clearly, this broad classification will depend on how strict the definition of a “good” bond is. In order to investigate the different level of quality requested by the market in different periods, we consider two different definitions of “good” bonds. In the first definition, that will be referred to as the *weak* definition of quality, we define “good” bonds as those with a rating between AAA and A and “bad” bonds the remaining ones. The second definition, that will be referred as the *strong* definition of quality, is more stringent since classified a bond as “good” only if the corresponding country has a AAA rating. Hence, for each time-window t , we define the indicator function $\mathbb{1}_{i \in \text{Good}}(t)$ as equal to one if bond i is rated in the “good” category in the time-window t and zero otherwise. Similarly, $\mathbb{1}_{i \in \text{Bad}}(t)$ is the indicator function that equals one if bond i is rated in the “bad” category in the time-window t and zero otherwise.

¹⁰Namely: AAA, AA, A, BBB, BB, B, CCC, CC, C, SD (selectively defaulted on some obligations). The first available bond price in the time stamp may correspond to a earlier date than the first available rating for the same bond (see Table 4). If this is the case, we attach to the bond the first available rating. The impact of this attribution is negligible since we divide the bonds into two aggregated categories according to two definitions of quality.

We then consider the four quantities

$$\begin{aligned}
\text{Good}_t^{\text{Buy}} &\equiv \frac{\sum_{j=1}^{N_t^E} \sum_{i=1}^{N_t^B} (E_j \implies B_i^{\text{Buy}})_t \mathbb{1}_{i \in \text{Good}}(t)}{N_t^E \sum_{i=1}^{N_t^B} \mathbb{1}_{i \in \text{Good}}(t)}, \\
\text{Good}_t^{\text{Sell}} &\equiv \frac{\sum_{j=1}^{N_t^E} \sum_{i=1}^{N_t^B} (E_j \implies B_i^{\text{Sell}})_t \mathbb{1}_{i \in \text{Good}}(t)}{N_t^E \sum_{i=1}^{N_t^B} \mathbb{1}_{i \in \text{Good}}(t)}, \\
\text{Bad}_t^{\text{Buy}} &\equiv \frac{\sum_{j=1}^{N_t^E} \sum_{i=1}^{N_t^B} (E_j \implies B_i^{\text{Buy}})_t \mathbb{1}_{i \in \text{Bad}}(t)}{N_t^E \sum_{i=1}^{N_t^B} \mathbb{1}_{i \in \text{Bad}}(t)}, \\
\text{Bad}_t^{\text{Sell}} &\equiv \frac{\sum_{j=1}^{N_t^E} \sum_{i=1}^{N_t^B} (E_j \implies B_i^{\text{Sell}})_t \mathbb{1}_{i \in \text{Bad}}(t)}{N_t^E \sum_{i=1}^{N_t^B} \mathbb{1}_{i \in \text{Bad}}(t)}.
\end{aligned} \tag{8}$$

For example, $\text{Good}_t^{\text{Buy}}$ is the average number of buy causal links hitting a bond of the category Good. These metrics can be interpreted as an index of distressed buying or distressed selling, depending on the kind of causal relationship adopted. Figure 3 shows their time series, distinguishing between the two definitions of quality, together with the corresponding levels under the null of no causality. The left and right columns of Figure 3 report, respectively, the average number of hits according to the first (weak) definition and to the second (strong) definition of quality.

It is interesting to note that, once we identify quality with top-rated AAA bonds (top-right panel), there is a striking increase of the index of distressed buying (black continuous line) toward good bonds during the 2007 financial crisis and the Eurozone crisis, whose inception dates back roughly around the beginning of 2009. This evidence is less pronounced when considering the weaker definition of quality. Moreover, there is scant evidence of distress selling (red dotted line in the top-right panel) in top rated AAA bonds in the second phase of the crisis (2010 and 2011) while we can observe a considerable increase on distress selling during the Eurozone crisis in all the other category (red dotted lines).

The implications of this novel empirical evidences are far reaching and they clearly show that prominent market players do chase for quality in periods of market distress but they do it in different ways: in the first phase, during the 2007–2008 financial crisis, they chased quality by buying sovereign debt bonds particularly of top-rated AAA quality but also of AA and A quality and without simultaneously liquidating massively other lower rating bonds. In fact only bad bonds in weak sense show a moderate distressed selling activity (bottom left panel) On the contrary, in the second phase, during the 2009 – 2011 Eurozone crisis, they required only top-quality sovereign debt, i.e. the flight-to-quality occurred exclusively toward AAA-rated sovereign bonds, also at the expense of other not AAA bonds who were simultaneously heavily liquidated.

Note that, the extraordinarily high number of hits toward the AAA-A category during the 2007-2008 financial crisis (which is manifested as the peak around mid-2008 of the black continuous curve in the top-right panel of Figure 3), is likely to be a consequence of a flight-to-quality from toxic assets (such as subprime mortgages) to highly rated sovereign debts. However, since our dataset does not include any of the assets that were perceived as toxic during the subprime mortgage crisis, we cannot fully investigate such type of flight-to-quality. Nevertheless, our analysis clearly points towards the existence of two different types of flight-to-quality: in the first one (during the 2007 – 2008 financial crisis) major banks rebalanced their portfolio from risky equity and structured asset to sovereign bonds, while in the second one (during the 2009 – 2011 Eurozone crisis) they rebalanced mainly their sovereign bond portfolio from low and medium-rated bonds to top-rated bonds. The latter could thus be termed a “flight-to-top-quality” event.

The hypothesis of occurrence of different flight-to-quality episodes suggested by the plots of Figure 3 can be statistically validated comparing the number of hits per category with those expected under the null in which links from equities to bond are randomly assigned. The computation of these expected values

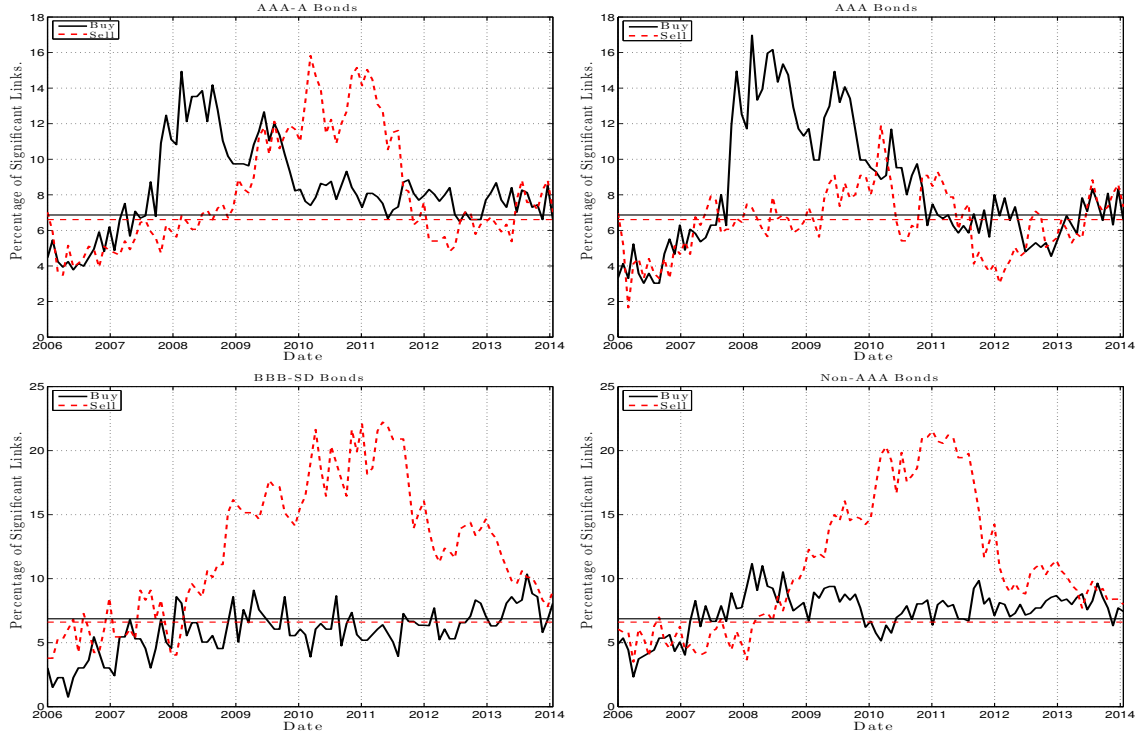


Figure 3: The four plots show the average number of causal links as defined in equation (8). The left column reports the results corresponding to the weak definition of bond quality while the right column corresponds to the strong one. In each plot the continuous black line refers to the average number of hits according to the number of $(E_j \implies B_i^{\text{Buy}})$ validated links, while the red dotted line is in correspondence of the number of $(E_j \implies B_i^{\text{Sell}})$ validated links. Horizontal lines are the corresponding levels under the null of no causality.

under the null is straightforward: if bonds were randomly hit by equities without any preference toward a specific category, then the probability to observe less than $N_{\text{Good},t}^{\text{Buy}}$ hits is given by the cumulative binomial distribution

$$P_{\text{Good},t}^{\text{Buy}} = \sum_{k=0}^{N_{\text{Good},t}^{\text{Buy}}} \binom{N_t^{\text{Buy}}}{k} p_{\text{Good},t}^k (1 - p_{\text{Good},t})^{N_t^{\text{Buy}} - k}, \quad (9)$$

where

$$p_{\text{Good},t} = \frac{\sum_{i=1}^{N_t^B} \mathbb{1}_{i \in \text{Good}}(t)}{N_t^B}$$

is the probability of having a good rated bonds at time t and where $N_t^{\text{Buy}} = \sum_{j=1}^{N_t^E} \sum_{i=1}^{N_t^B} (E_j \implies B_i^{\text{Buy}})_t$, is the total number of significant $(E_j \implies B_i^{\text{Buy}})_t$ links. The computation of $P_{\text{Bad},t}^{\text{Sell}}$ and the other probabilities follows the same rule.

When the probability in (9) is larger than a threshold p (e.g. $p = 99\%$), it means that there is a statistically significant number of $(E_j \implies B_i^{\text{Buy}})_t$ casual links from equities to good rated bonds. A similar reasoning is valid for the bad rated ones and $(E_j \implies B_i^{\text{Sell}})_t$ casual links. A flight-to-quality episode can hence be identified when both $P_{\text{Good},t}^{\text{Buy}}$ and $P_{\text{Bad},t}^{\text{Sell}}$ are larger than p simultaneously. Results of the flight-to-quality test are reported in Figure 4 distinguishing, as before, between the two definitions of good and bad bonds. The left panel corresponds to the weak definition where good bonds are those laying in the class from AAA to A, while the right panel corresponds to the strong definition of quality where a bond is classified as good only if the country has a AAA rating. The vertical thick lines are in correspondence of the periods in which our test identifies a flight-to-quality. They are distinguished into three cases, continuous black vertical

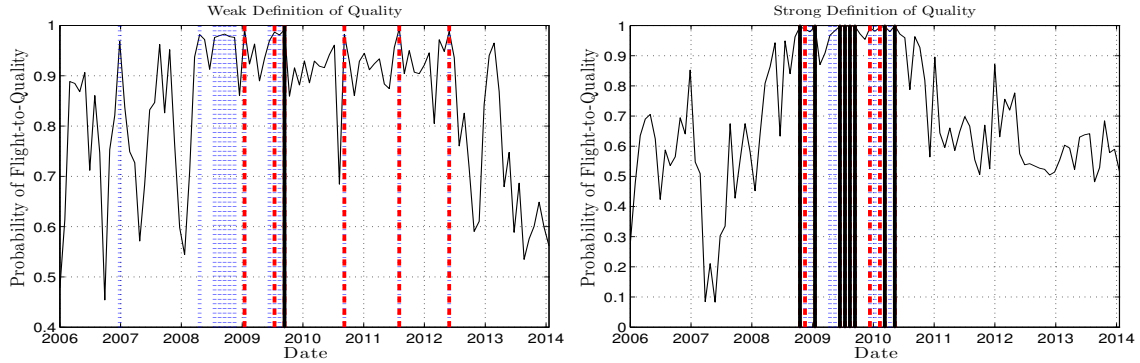


Figure 4: The figure shows the proposed flight-to-quality measure. Vertical lines are in correspondence of time-windows in which both $P_{\text{Good},t}^{\text{Buy}}$ and $P_{\text{Bad},t}^{\text{Sell}}$ are above a threshold p , which is fixed to 99% for the continuous black vertical lines, to 97.5% for the thick red dotted lines and to 95% for thin dotted blue lines. The thin black curve represents the quantity $(P_{\text{Good},t}^{\text{Buy}} + P_{\text{Bad},t}^{\text{Sell}})/2$, that is the arithmetic mean of the two probabilities.

lines correspond to $p = 99\%$, thick red dotted ones to $p = 97.5\%$ and thin dotted blue lines to $p = 95\%$.

The scenario depicted by the results of Figure 4 confirms the intuition that we got from the preliminary analysis inspired by the plots in Figure 3 and from the network snapshots of Figure 1: quality is required as consequence of the turmoil and distrust spread by the Eurozone crisis, and the phenomenon is well-identified if quality is defined by top-quality AAA-rated bonds. In fact, with the weak definition of quality, we find only one highly significant period of flight-to-quality after mid-2009 and many other less significant scattered all around the time stamp. On the contrary, the strong definition gives a concentrated sequence of highly significant events around the Eurozone crisis and even before the beginning of 2009. In each panel of Figure 4 we add a black curve in correspondence of the arithmetic mean of the two probabilities $P_{\text{Good},t}^{\text{Buy}}$ and $P_{\text{Bad},t}^{\text{Sell}}$. This quantity may be thought as an early indicator of flight-to-quality. In fact, while the weak definition of quality returns a quite noisy path for such an indicator, in the strong definition we observe a much more smooth curve. In particular, in the case of the right panel, the indicator starts increasing in mid-2007, much before the onset of the Eurozone crisis, and peaks some months before 2009. This result could be conjectured to be a consequence of the capability of big market player of fearing the effect of the 2007 financial crisis on the the sovereign debt of countries with a weak fiscal discipline, although a deeper analysis is required to confirm such an hypothesis.

7 Out-of-sample analysis

In order to test the predictive power of the Granger analysis we perform out-of-sample regressions which, in spirit, follow the approach proposed in Billio et al. (2012). For a given time-window t and a given sovereign bond i , we define the three centrality measures

$$H_{i,t}^{\text{Sell}} \equiv \frac{1}{N_t^E} \sum_{j=1}^{N_t^E} (E_j \implies B_i^{\text{Sell}})_t, \quad H_{i,t}^{\text{Buy}} \equiv \frac{1}{N_t^E} \sum_{j=1}^{N_t^E} (E_j \implies B_i^{\text{Buy}})_t, \quad H_{i,t} \equiv \frac{1}{N_t^E} \sum_{j=1}^{N_t^E} (E_j \implies B_i)_t, \quad (10)$$

which, in network terms, are the degree of the bond nodes. The economic interpretation of the quantities defined above is straightforward. For example $H_{i,t}^{\text{Sell}}$ is the percentage of significant causal links ($E_j \implies B_i^{\text{Sell}}$) coming from the equity side of the network that hit the i -th bond in the time-window t . Large

values of $H_{i,t}^{\text{Sell}}$ are expected when the i -th bond is subject to a large amount of distressed selling. Thus, a similar interpretation is valid for $H_{i,t}^{\text{Buy}}$, although the economic interpretation of $H_{i,t}$ is less direct, since the Granger (1969) captures causality in mean. Nevertheless, in our out-of-sample forecast exercise, we take into consideration $H_{i,t}$ as well for comparison.

7.1 Dynamic proxies of quality

In this section we adopt and describe four proxies of sovereign bond quality that are defined “dynamically” in the sense of a continuous, time-dependent, real variables, in opposition to agency ratings that are piecewise constant categorical variables. A finer description of the bond quality is needed since the the idea is to test the forecasting power of suitable centrality measures of the causality networks in predicting the bond quality. For this purpose agency ratings are not particularly suited mainly because they are almost constant during time and, when they change, they signal a huge downgrade or, more rarely, a huge upgrade of the sovereign debt quality.

Our conjecture is that, during periods in which systemic events are in place, centrality measure play an import an role in forecasting the future bond quality. This would corroborate their role as early warning indicators of systemic risk.

The first proxy is based on the correlation between the series of sovereign bond yields and the corresponding credit default swap of a given country. A simple absence of arbitrage argument links the spread \mathcal{S} of CDS with the par-yield γ of a bond of the same entity. As pointed out by Hull et al. (2004), the difference between a CDS spread and the corresponding par-yield should equal the risk-free rate, or

$$\mathcal{S} = \gamma - r, \quad (11)$$

The difference between a CDS spread and the excess par-yield over the risk-free rate, $b = \mathcal{S} - (\gamma - r)$, is usually referred as the basis. Hence, the no-arbitrage condition requires the basis to be zero. On a large sample of companies and sovereign data, Hull et al. (2004) find that the no-arbitrage relationship between CDS and par-yield holds fairly well, with a risk-free rate slightly smaller than the swap rate and above the Treasury rate. In our dataset¹¹ we observe that, especially in periods of financial distress, such arbitrage relationship is often violated, particularly for those countries whose credit quality is commonly perceived as remarkably good.

Figure 5 visualizes this phenomenon. The left and right panels report the five-years maturity CDS spread (continuous black line) and the corresponding five-years maturity sovereign bond yield (dotted red line) for Russia (left) and Germany (right). The bond-yield reported is the nominal par-yield after the subtraction of the zero-coupon curve, whose computation is mentioned in Section 4, and hence it should equal the CDS spread according to the no-arbitrage condition (11). The period depicted includes the Eurozone crisis. The plots in Figure 5 put in evidence that, while Russia has a CDS-bond correlation of almost 100%, in agreement with the no-arbitrage constraint, the case of Germany is totally different, showing even a negative CDS-bond correlation. Intuitively, CDS spreads increase during a period of crisis since investors require higher premium, however the behavior of bond yield can be very different according to the perceived credit quality of the country. The case of Germany is the most striking: large capital flows are directed toward the German sovereign debt pushing downward the level of the corresponding yield, while the CDS spread continues its rise as a consequence of the generalized increase in global credit risk. This empirical evidence has been somehow foreseen by the past literature. In fact, price gaps among securities with identical cash flow have been theoretically justified by the general equilibrium model by Gârleanu and Pedersen (2011), where the surge the CDS-bond basis is due to negative shocks to the economy that force agents to hit their

¹¹Our dataset has almost zero overlap on time with that of Hull et al. (2004).

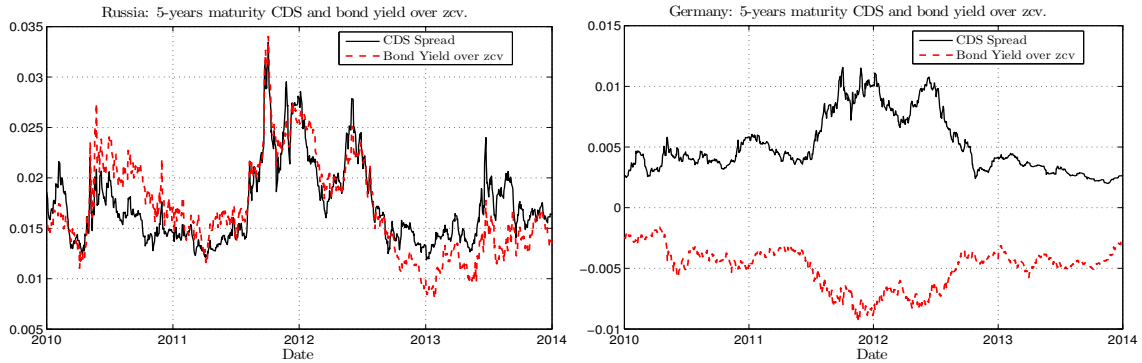


Figure 5: Left and right panels report, respectively, the five-years maturity spread (continuous black line) and bond yield over the zero-coupon curve (dotted red line) for Russia and Germany in the period that starts in January, 1, 2007 and ends at February, 14, 2014.

their margin constraints. Moreover the formation of CDS-bond bases has been recently addressed as the consequence of flight-to-quality episodes (Fontana and Scheicher, 2010).

In our analysis we do not investigate the determinants of CDS-bond bases, limiting ourselves to the ranking of bond quality obtained by using the CDS-bond correlation. In the light of the empirical evidences of Figure 5 and the most recent interpretation of the CDS-bond joint dynamics, we introduce our first dynamical proxy for quality as the one minus the CDS-bond correlation, i.e.

$$C_i = 1 - \text{Corr}[\Delta\gamma_{i,t}, \Delta\mathcal{S}_{i,t}], \quad C_i \in [0, 2], \quad (12)$$

where the capital letter C is introduced to remind that the quality proxy is based on the CDS-bond correlation¹² coefficient and where $\Delta\gamma_{i,t}$ and $\Delta\mathcal{S}_{i,t}$ indicates, respectively, the series of one-lag difference for $\gamma_{i,t}$ and $\mathcal{S}_{i,t}$.

The first column of Table 1 reports the country ranking according to the quality measure C_i , computed using all observations after the beginning of 2007. As anticipated we observe a clear alignment between the country and what it is intuitively expected: countries with weak fiscal discipline (e.g. Spain, Italy and Portugal) stay in the bottommost part of the table with very low values of C_i , while the largest values are reached for countries such as Germany or Austria, whose claims on sovereign debts are undoubtedly perceived as more reliable. The case of Greece is not particularly significant since the corresponding CDS series show a very peculiar behavior, with a diverging dynamics during the Eurozone crisis.

In order to have a multifaceted description of the bond quality, we introduce three further measures that are obtained exclusively using the bond yields time series. The simplest univariate measure related to bond quality is the bond yield “realized volatility”, defined as

$$RV_i = \sum_{t \geq 2007} (\Delta\gamma_{i,t})^2,$$

where $\gamma_{i,t}$ indicates the daily series of the nominal bond yield for country i . The larger the value of RV_i the greater the uncertainty about its value, hence we expect RV_i to be inversely related to bond quality. This intuition is confirmed by the second column of Table 1, where countries are ranked according to increasing values of RV_i , with Japan being the less volatile and Greece at the end of the ranking.

The remaining two measures are based on yield spreads defined in the standard way, i.e. the difference

¹²Note that in equation (12) we are not specifying which time period is used for the two series $\Delta\gamma_{i,t}$ and $\Delta\mathcal{S}_{i,t}$, nevertheless this will become clear according to the specific case.

between a given sovereign bond yield and the corresponding German bond yield at the same maturity (which is, in our applications, five years). The introduction of yield spreads in our analysis is justified by the vast literature on the role of spreads in many aspects of the global economy. For example, the prominent role of spreads in the European Monetary Union as a source of risk is analyzed, much before the Eurozone crises, by Geyer et al. (2004) with the adoption of a two-factors model. Moreover, Manganelli and Wolswijk (2009) investigate the relationship between macro-economic policies and yield spreads. Their empirical findings confirm that changes in interest rates affect the risk-aversion of investors producing a significant impact on yield spreads. Global spreads and fiscal fundamentals of the euro area are used to proxy expected exchange rate devaluation in an econometric model for euro area spread by Favero (2013). De Santis (2014) identifies a flight-to-liquidity premium as the only factor explaining the sovereign spreads for countries with low credit risk such as Netherlands and Finland.

The yield spread is formally defined as $s_{i,t} = \gamma_{i,t} - \gamma_{\text{GER},t}$, where $\gamma_{\text{GER},t}$ is the bond yield for Germany. The spread-based measures that we adopt are defined as

$$\bar{S}_i = \frac{1}{N_i} \sum_{t \geq 2007} s_{i,t}, \quad \text{and} \quad \text{SM}_i = \max_{t \geq 2007} s_{i,t},$$

which represent, respectively, the average and the maximum spread in the period after January, 1, 2007. Note that, as for RV_i , the spread-based measures are expected to be inversely related to quality. This is confirmed by the third and fourth columns of Table 1 where, typically, we observe countries like Greece or Portugal with the highest values of the spread-based measures and Switzerland, Singapore and Netherlands among the lowest ones¹³.

We adapt the definitions of the four quality proxies introduced above in order to perform genuine out-of-sample forecasts. For this purpose, we dynamically compute the quality proxies over a moving windows $(t, t+h]$ with h the number of days ahead over which the forecast is performed. Then, the four quality proxies are dynamically restated as

$$\begin{aligned} C_{i,t+h} &= 1 - \text{Corr} [\Delta\gamma_i, \Delta S_i]_{t:t+h}, \\ \text{RV}_{i,t+h} &= \sum_{\tau=t}^{t+h} (\Delta\gamma_{i,\tau})^2, \\ \bar{S}_{i,t+h} &= \frac{1}{h} \sum_{\tau=t}^{t+h} s_{i,\tau}, \\ \text{SM}_{i,t+h} &= \max_{\tau \in (t,t+h]} s_{i,\tau}. \end{aligned} \tag{13}$$

Similarly to what it is done in Billio et al. (2012), in order to mitigate the effect of outliers (see, for example, the case of Greece in Table 1), we switch to the rankings of those measures introducing calligraphic notation for all the variables involved. For example $\mathcal{H}_{i,t}^{\text{Sell}}$ indicates the ranking of $H_{i,t}^{\text{Sell}}$ in the t -th time-window¹⁴, $\bar{S}_{i,t}^H$ will indicate the ranking of $S_{i,t}^H$ in $(t, t+h]$, and so on.

7.2 Out-of-sample cross-sectional regressions

The out-of-sample regressions that we run are designed to test if, especially in periods of distress, the cross-sectional rankings of bond quality proxies can be predicted by past bond centrality measures. Since each of the dynamical proxies of quality is expected to be persistent (or strongly persistent in the case of realized volatility) it is important, to avoid spurious results, to include the lagged value of the dependent variable

¹³Germany has trivially a zero value for all these spread-based measures.

¹⁴Hence $\mathcal{H}_{i,t}^{\text{Sell}} \approx 1$ for the bond with the highest value of $H_{i,t}^{\text{Sell}}$ and $\mathcal{H}_{i,t}^{\text{Sell}} \approx 0$ for that with the lowest value.

Table 1: Ranking of country according to quality proxies.

C_i		RV_i		\bar{S}_i		SM_i	
1.275	Germany	0.933	Japan	-2.453	Switzerland	-1.054	Singapore
1.149	Australia	1.885	Switzerland	-2.242	Singapore	-0.855	Switzerland
1.115	Denmark	3.119	Singapore	-1.828	Hong Kong	0.000	Germany
1.113	Sweden	3.832	New Zealand	-1.531	Japan	0.008	Japan
1.111	Romania	4.264	Sweden	-1.241	United States	0.563	Hong Kong
1.075	Czech Rep.	4.390	Finland	-1.199	Finland	0.861	Netherlands
1.069	United Kingdom	4.475	Hong Kong	-1.085	France	1.201	Denmark
1.062	Norway	4.643	Netherlands	-0.785	Sweden	1.341	United Kingdom
1.042	United States	4.797	Denmark	-0.130	Canada	1.454	Sweden
1.029	Japan	4.849	Canada	-0.100	Malaysia	1.733	Norway
1.025	Singapore	4.886	Norway	0.000	Germany	1.910	Canada
1.004	Bulgaria	4.911	France	0.016	Slovakia	2.080	Austria
0.993	New Zealand	4.964	Poland	0.178	Denmark	2.095	United States
0.976	Finland	5.079	Czech Rep.	0.208	Italy	2.115	Finland
0.967	Hong Kong	5.105	Germany	0.238	Netherlands	2.129	France
0.959	Canada	5.170	United Kingdom	0.404	Austria	2.374	Slovakia
0.944	Switzerland	5.171	Austria	0.509	United Kingdom	2.411	Czech Rep.
0.937	Malaysia	6.325	Croatia	0.588	Poland	2.622	Malaysia
0.936	Turkey	6.442	Belgium	0.700	Bulgaria	3.097	New Zealand
0.885	Netherlands	7.187	Slovakia	0.700	Belgium	3.121	Poland
0.880	Slovakia	8.343	Australia	0.767	Czech Rep.	3.703	Italy
0.846	Hungary	8.436	United States	0.874	Norway	3.792	Australia
0.777	Poland	9.072	Malaysia	1.012	New Zealand	4.103	Belgium
0.746	Greece	10.803	Slovenia	1.680	Spain	5.044	Bulgaria
0.705	Austria	14.926	Italy	1.854	Slovenia	5.640	Slovenia
0.645	Croatia	15.161	Spain	2.300	Portugal	6.561	Croatia
0.642	Cyprus	21.824	Hungary	2.383	Russia	6.705	Russia
0.591	France	23.646	Russia	2.415	Australia	6.981	Spain
0.553	Slovenia	27.438	Ireland	2.484	Ireland	8.790	Hungary
0.544	Russia	68.692	Portugal	2.849	Hungary	9.939	Romania
0.369	Belgium	70.947	Venezuela	2.934	Croatia	14.043	Ireland
0.339	Ireland	71.989	Romania	3.940	Cyprus	15.454	Cyprus
0.337	Venezuela	85.221	Turkey	3.994	Romania	16.812	Portugal
0.327	Portugal	88.153	Bulgaria	7.974	Turkey	18.306	Venezuela
0.249	Italy	108.341	Cyprus	9.040	Venezuela	19.843	Turkey
0.220	Spain	846.110	Greece	11.406	Greece	55.484	Greece

Note: Each of the four panels contains the ranking of all countries according to the corresponding variable reported in the column label. Each variable is computed using all data available after 2007. More precisely, C_i is defined as $1 - \rho_i$ where ρ_i is the sample correlation between the sovereign bond yield and the CDS spread (and it is plotted in decreasing order). RV_i is the realized volatility of bond yield variations. \bar{S}_i and SM_i are, respectively mean and maximum value of the spread between the bond of the i -th country and the German bond.

in the forecasting regression. It is also important to stress that, at any point in time, the regressors (i.e. the centrality measures (10) and the lagged dependent variable) and the dependent variables (one of the quality proxies (13)) are computed over time windows having zero overlaps. More precisely, the regressors are computed on a past time horizon that is $(t-h, t]$ for the lagged dependent variable and that is the three years preceding t for the network centrality measures. The regressors are indicated with the subscript t , while the chosen dependent variable (i.e. one among the four quality proxies) is calculated over the forecasting period $(t, t+h]$ and denoted with the subscript $t+h$.

In our analysis we adopt a twofold strategy. First, for every t , we run the two nested cross-sectional regressions

$$\mathcal{Y}_{i,t+h} = K^{(1)} + \alpha_t \mathcal{Y}_{i,t} + \varepsilon_i^{(1)} \quad (14)$$

and

$$\mathcal{Y}_{i,t+h} = K^{(2)} + \alpha_t \mathcal{Y}_{i,t} + \beta_t \mathcal{X}_{i,t} + \varepsilon_i^{(2)}, \quad (15)$$

where $\mathcal{Y}_{i,t}$ is the ranking of one among $C_{i,t}$, $RV_{i,t}$, $\bar{S}_{i,t}$ and $SM_{i,t}$ defined in (13), while the independent variable $\mathcal{X}_{i,t}$ is the ranking of one among the centrality measures defined in (10).

In the setting of equations (14)-(15) we are testing whether or not the addition of the systemically relevant variable is improving the simple auto-regression (14). Figures 6, 7 and 8 report the t-statistic of the regression coefficient β_t in (15) for the regressions with dependent variable, respectively, $C_{i,t}$, $\mathcal{R}V_{i,t}$ and¹⁵ $SM_{i,t}$. We choose a value of $h = 360$, which corresponds to annual forecasts¹⁶. Black points are in correspondence of the regressions in which, according to a standard F-test with 95% confidence, the addition of $\mathcal{X}_{i,t}$ significantly improves the simple auto-regressive model (14).

The second approach consists in comparing, for a given proxy of the i -th bond quality $\mathcal{Y}_{i,t+h}$ computed in the forecasting horizon $(t, t+h]$, the forecasting ability of past values of the dependent variable itself against that of systemic risk variables. For a given t we then compare the cross-sectional regression (14) with a new regression where the lagged dependent variable is substituted by one of the systemic relevant variables, that is

$$\mathcal{Y}_{i,t+h} = K^{(2)'} + \beta'_t \mathcal{X}_{i,t} + \varepsilon_i^{(2)'} \quad (16)$$

For this regression we compute the t-statistics of β'_t and the p-value of the Vuong (1989) likelihood ratio test¹⁷. Since, as mentioned before, the dependent variables $\mathcal{Y}_{i,t}$ are typically persistent, we expect that only in some peculiar periods the systemic relevant variates over-perform the past values of $\mathcal{Y}_{i,t}$ in forecasting future values of $\mathcal{Y}_{i,t+h}$.

7.3 Results

The regression results are depicted in Figures from 6 to 9¹⁸ and can be summarized in three main empirical findings.

First, the black points in Figure 6 shows a significant negative impact of $\mathcal{H}_{i,t}^{\text{Sell}}$ on future values of CDS-bond correlation rankings. This means that the bond quality ranked according to C_i is, in period of financial distress, significantly influenced by the intensity of distressed selling experienced by the bond in the past year. The negative sign is in agreement with the idea that high values of C_i correspond to a better quality

¹⁵For the sake of exposition, results relative to $\bar{S}_{i,t}$ are reported in Figures 11 of the Appendix F.

¹⁶Results with different h are similar and available upon request.

¹⁷The test is two-sided and it is designed to be around 100% when model (14) outperforms model (16) and around 0% the other way round.

¹⁸For the second regression (16) we only report the results obtained when the dependent variable is the quality proxy based on bond-CDS correlation.

Nested models comparison with dependent variable $C_{i,t}$.

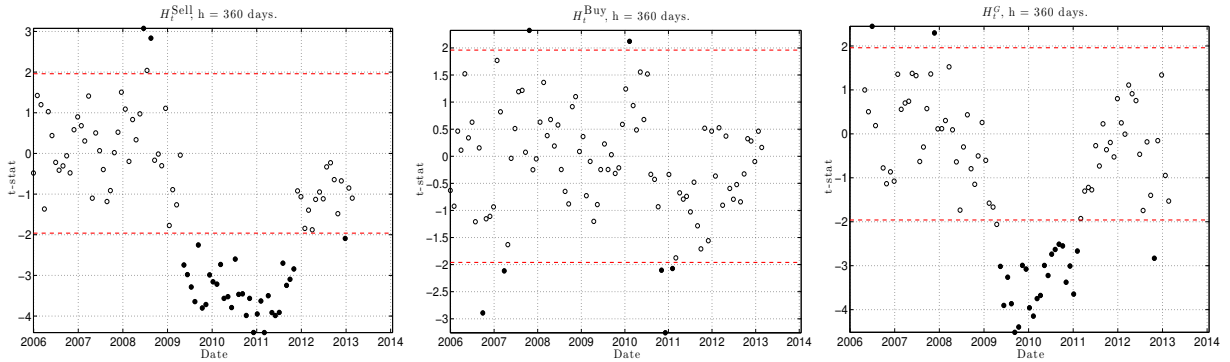


Figure 6: This figure depicts, for each time-window $t = 1, \dots, 99$ whose closure is at day e_t (reported in the horizontal axis), the t-statistic of β_t in (15) where the dependent variable $\mathcal{Y}_{i,t}$ is the ranking of the variable $C_{i,t}^H$ defined in (13) and computed in the forecasting horizon $(e_t, e_t + h]$, with $h = 360$. Each column corresponds to a different regressor $\mathcal{X}_{i,t}$ in (15). More precisely, $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}^{B+}$ for the first, $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}^{B-}$ for the second and $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}$ for the third. A black point is reported whenever the regression (15) is significantly better (according to a standard F-test with 95% confidence) than the simple auto-regressive model (14). Horizontal red dotted lines are in correspondence of 95% confidence level of the standard Normal distribution.

for the bond (see Table 1). On the contrary, the distressed buying indicator has no strong influence on future value of C_i , besides the direction of the impact is also slightly noisy. Note that, as for the results in Figure 2, the regression with the original Granger (1969)-based measure of centrality are roughly in agreement with those of the distressed selling indicator. The non-nested analysis reported in Figure 9 corroborates the results of Figure 6, with the intriguing implication that, during periods of financial turmoil, systemic variables are more informative than past values of quality measures in predicting the future quality of the bonds.

Second, the realized volatility of bond yield is positively impacted by the indicator of distressed selling $\mathcal{H}_{i,t}^{\text{Sell}}$ and by the Granger (1969)-based measure of centrality, and this is particularly true for $\mathcal{H}_{i,t}^{\text{Sell}}$ during the Eurozone crisis. The distressed buying indicator $\mathcal{H}_{i,t}^{\text{Buy}}$ shows a noisier behavior, even if it tends to show a negative sign when significant (as intuitively expected) over most of the sample (turning positive only in the final periods).

Third, the maximum spread is largely influenced by $\mathcal{H}_{i,t}^{\text{Sell}}$ for the entire period of the Eurozone crisis (we remember that each point of the horizontal axis refers to the previous three years for the dependent network centrality, and to the subsequent year for the dependent quality proxy) and even later.

In summary, we can assert that the percentage of significant $(E_j \implies B_i^{\text{Sell}})$ and $(E_j \implies B_i)$ causal links has a clear impact on future values of the bond quality proxies, while the distress buying indicator may go in different directions depending on the period analyzed.

Nested models comparison with dependent variable $\mathcal{RV}_{i,t}$.

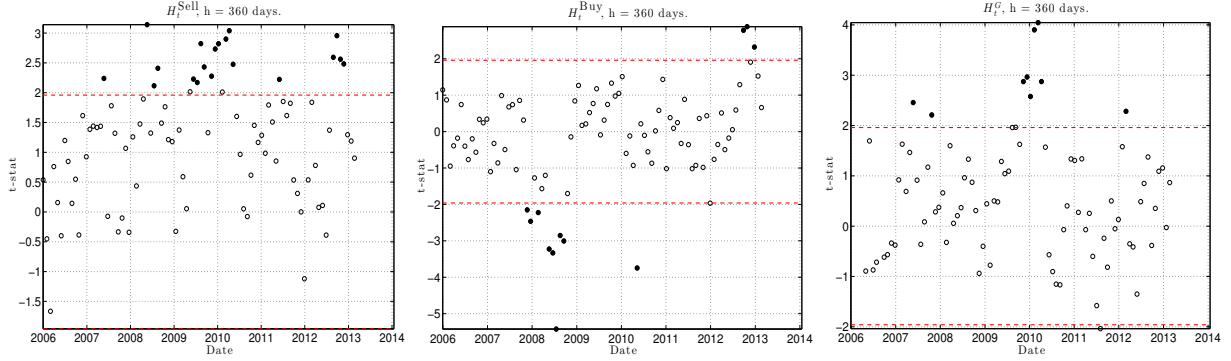


Figure 7: This figure depicts, for each time-window $t = 1, \dots, 99$ whose closure is at day e_t (reported in the horizontal axis), the t-statistic of β_t in (15) where the dependent variable $\mathcal{Y}_{i,t}$ is the ranking of the variable $\mathcal{RV}_{i,t}^H$ defined in (13) and computed in the forecasting horizon $(e_t, e_t + h]$, with $h = 360$. Each column corresponds to a different regressor $\mathcal{X}_{i,t}$ in (15). More precisely, $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}^{B+}$ for the first, $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}^{B-}$ for the second and $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}$ for the third. A black point is reported whenever the regression (15) is significantly better (according to a standard F-test with 95% confidence) than the simple auto-regressive model (14). Horizontal red dotted lines are in correspondence of 95% confidence level of the standard Normal distribution.

Nested models comparison with dependent variable $\mathcal{SM}_{i,t}$.

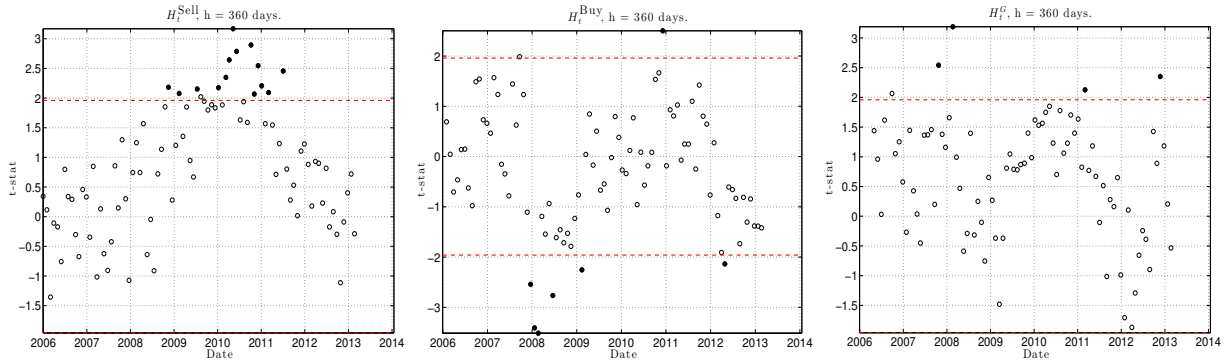


Figure 8: This figure depicts, for each time-window $t = 1, \dots, 99$ whose closure is at day e_t (reported in the horizontal axis), the t-statistic of β_t in (15) where the dependent variable $\mathcal{Y}_{i,t}$ is the ranking of the variable $\mathcal{SM}_{i,t}^H$ defined in (13) and computed in the forecasting horizon $(e_t, e_t + h]$, with $h = 360$. Each column corresponds to a different regressor $\mathcal{X}_{i,t}$ in (15). More precisely, $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}^{B+}$ for the first, $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}^{B-}$ for the second and $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}$ for the third. A black point is reported whenever the regression (15) is significantly better (according to a standard F-test with 95% confidence) than the simple auto-regressive model (14). Horizontal red dotted lines are in correspondence of 95% confidence level of the standard Normal distribution.

Non-nested models comparison with dependent variable $\mathcal{C}_{i,t}$.

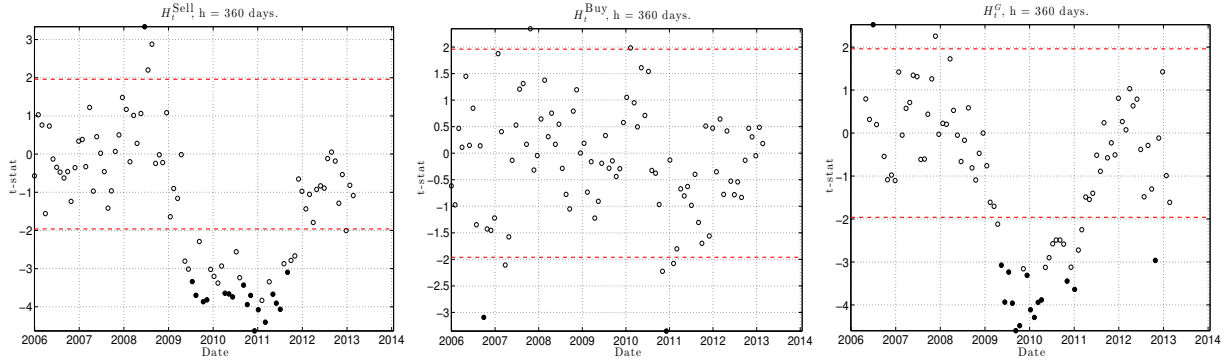


Figure 9: This figure depicts, for each time-window $t = 1, \dots, 99$ whose closure is at day e_t (reported in the horizontal axis), the t-statistic of β'_t in (16) where the dependent variable $\mathcal{Y}_{i,t}$ is the ranking of the variable $\mathcal{C}_{i,t}^H$ defined in (13) and computed in the forecasting horizon $(e_t, e_t + h]$, with $h = 360$. Each column corresponds to a different regressor $\mathcal{X}_{i,t}$ in (16). More precisely, $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}^{B+}$ for the first, $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}^{B-}$ for the second and $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}$ for the third. A black point is reported whenever the regression (16) is significantly better (according to the log-likelihood ratio test of Vuong (1989) with 95% confidence) than the simple auto-regressive model (14). Horizontal red dotted lines are in correspondence of 95% confidence level of the standard Normal distribution.

8 Conclusions

In this paper we introduce an econometric method to detect and analyze events of financial distress. It is designed to test if two variables are Granger-connected in the tails of their distributions, hence it is chiefly based on events of extraordinary nature. We derive Granger centrality measures of risk spillover between equity log-returns of 33 systemically relevant banks and government bond yield variations for 36 countries around the world. Moreover, we give a simple economic interpretation of such centrality measures in terms of indicators of distressed selling and distressed buying.

Exploiting the information of S&P country ratings, our empirical analysis evidences that, during the turbulent period of the Eurozone crisis, major banks across the world significantly impacted the sovereign bond market chasing for quality. More specifically, they looked for top-quality bonds, freeing capitals from the large part of non-AAA-rated bonds and moving them toward the AAA-rated ones. Besides, an intense buying activity of AAA-rated bonds is certified during the subprime mortgages crisis. Although, we do not have specific information on assets which were classified as toxic during the 2007 financial crisis, it is very likely that the surge of the distressed buying index for AAA-rated bonds is due to flight-to-quality episodes from the stock market to the government bond market. Adopting different dynamic measures of bond quality allow us to test the out-of-sample forecast performances of the centrality measures. The indicator of distressed selling has significant explanatory power in forecasting future values of the correlation between CDS spreads and the bond yield of a country, of the yield realized volatility and of the maximum spread with respect to the German bond. The behavior of the centrality measure based on the original Granger (1969) causality, although with a reduced forecasting power, is in line with the indicator of distressed selling. Finally, the results of the analysis suggest the possibility of adopting the Hong et al. (2009) and Granger (1969) causality measures as early warning indicators of systemic risk. Our conjecture is that the red alarm should be switched on every time that the information contained in the network is able to improve the forecast of bond quality measures, or even when it turns out to be a better regressor than the past values of the quality measure itself. Although this appears as an intriguing application of the proposed methodology, it requires further verifications possibly with the adoption of high-frequency data and it is postponed for future research.

A Profit maximization with two assets.

In this Appendix we prove Proposition 1 stated in Section 2. To ease the readability, we report the constrained portfolio optimization below.

$$\begin{aligned} \max_{A, \omega} \quad & A \boldsymbol{\mu}' \boldsymbol{\omega} \\ \text{s.t.} \quad & \alpha A \sqrt{\boldsymbol{\omega}' \boldsymbol{\Sigma} \boldsymbol{\omega}} \leq E. \end{aligned} \tag{17}$$

Before proceeding, we need to prove the following

Lemma. *The optimal solution of the optimization problem (17) is given by*

$$\begin{aligned} \omega^* &= \frac{\mu_a \sigma_b^2 - \mu_b \sigma_{ab}}{\mu_a (\sigma_b^2 - \sigma_{ab}) + \mu_b (\sigma_a^2 - \sigma_{ab})}, \\ A^* &= \frac{E}{\alpha \sqrt{\boldsymbol{\omega}^{*'} \boldsymbol{\Sigma} \boldsymbol{\omega}^*}}. \end{aligned}$$

Proof.

First consider that the VaR constraint

$$\alpha A \sqrt{\boldsymbol{\omega}' \boldsymbol{\Sigma} \boldsymbol{\omega}} \leq E$$

always binds since there is no profit for the investor in leaving part of its equity uninvested. The Lagrangian function to be maximized is thus

$$\mathcal{L}(A, \omega, \gamma) = A \boldsymbol{\mu}' \boldsymbol{\omega} - \frac{\gamma}{2} [\alpha^2 A^2 \boldsymbol{\omega}' \boldsymbol{\Sigma} \boldsymbol{\omega} - E^2],$$

where γ is a Lagrange multiplier. The first-order condition associated to the total asset is written as

$$\frac{\partial \mathcal{L}(A, \omega, \gamma)}{\partial A} = \boldsymbol{\mu}' \boldsymbol{\omega} - \gamma \alpha^2 A \boldsymbol{\omega}' \boldsymbol{\Sigma} \boldsymbol{\omega} = 0,$$

whose solution is

$$A = \frac{\boldsymbol{\mu}' \boldsymbol{\omega}}{\gamma \alpha^2 \boldsymbol{\omega}' \boldsymbol{\Sigma} \boldsymbol{\omega}}.$$

Putting the equation above in the VaR constraint returns the optimal γ as

$$\gamma = \frac{\boldsymbol{\mu}' \boldsymbol{\omega}}{\alpha E \sqrt{\boldsymbol{\omega}' \boldsymbol{\Sigma} \boldsymbol{\omega}}}. \tag{18}$$

The remaining first order condition is that relative to ω and it is written as

$$\frac{\partial \mathcal{L}(A, \omega, \gamma)}{\partial \omega} = A (\mu_a - \mu_b) - \gamma \alpha^2 A^2 [\omega (\sigma_a^2 + \sigma_b^2 - 2\sigma_{ab}) + \sigma_{ab} - \sigma_b^2] = 0. \tag{19}$$

Using the constraint $A = E/(\alpha \sqrt{\boldsymbol{\omega}' \boldsymbol{\Sigma} \boldsymbol{\omega}})$ and equation (18) the condition in (19) becomes

$$(\mu_a - \mu_b) \boldsymbol{\omega}' \boldsymbol{\Sigma} \boldsymbol{\omega} - \boldsymbol{\mu}' \boldsymbol{\omega} [\omega (\sigma_a^2 + \sigma_b^2 - 2\sigma_{ab}) + \sigma_{ab} - \sigma_b^2] = 0.$$

Despite the appearance of the quadratic form $\boldsymbol{\omega}' \boldsymbol{\Sigma} \boldsymbol{\omega}$, the previous equation is of degree one in ω and gives as an optimal solution

$$\omega^* = \frac{\mu_a \sigma_b^2 - \mu_b \sigma_{ab}}{\mu_a (\sigma_b^2 - \sigma_{ab}) + \mu_b (\sigma_a^2 - \sigma_{ab})}. \tag{20}$$

□

We can now proof the proposition stated in the main text, which is reported here to ease readability.

Proposition. *A profit-maximizer that allocates the available resources according to (17) always reacts to equity drops with a flight-to-quality. In formula*

$$\frac{d\omega}{dE} < 0.$$

Proof.

Suppose that the equity is perturbed $E \rightarrow E + dE$. If the debt is approximately constant (see Adrian and Shin, 2010) then the variation of the total asset equals the variation of the total equity, hence $dA = dE$. The perturbation of the VaR constraint

$$\alpha \sqrt{\omega' \Sigma \omega} = \frac{E}{A}$$

gives

$$\frac{\alpha}{2 \sqrt{\omega' \Sigma \omega}} \frac{d(\omega' \Sigma \omega)}{d\omega} d\omega = \frac{dE}{A} - \frac{E}{A^2} dE = x \frac{dE}{\lambda} \left(1 - \frac{1}{\lambda}\right),$$

where we have used $dA = dE$, x is the percentage equity shock $dE = x E$ and where $\lambda = \frac{A}{E} > 1$ is the optimal leverage. It is immediate to see that

$$\frac{1}{2} \frac{d(\omega' \Sigma \omega)}{d\omega} = \omega (\sigma_a^2 + \sigma_b^2 - 2 \sigma_{ab}) + \sigma_{ab} - \sigma_b^2,$$

whence

$$d\omega = x \frac{dE}{\alpha \lambda} \left(1 - \frac{1}{\lambda}\right) \frac{\sqrt{\omega' \Sigma \omega}}{\omega (\sigma_a^2 + \sigma_b^2 - 2 \sigma_{ab}) + \sigma_{ab} - \sigma_b^2} \quad (21)$$

Since $\lambda > 1$ the sign of $d\omega$ is equal or opposite to that of dE according whether the quantity

$$\omega (\sigma_a^2 + \sigma_b^2 - 2 \sigma_{ab}) + \sigma_{ab} - \sigma_b^2$$

is positive or negative. We show that the second case is the only one which is compatible with the optimality. Re-write the optimal ω^* in (20) as

$$\omega^* = \frac{\sigma_b^2 - \sigma_{ab} - \frac{\mu_b - \mu_a}{\mu_a} \sigma_{ab}}{\sigma_a^2 + \sigma_b^2 - 2 \sigma_{ab} + \frac{\mu_b - \mu_a}{\mu_a} (\sigma_a^2 - \sigma_{ab})}$$

Since risk premia are positive we have $\frac{\mu_b - \mu_a}{\mu_a} > 0$ and hence

$$\omega^* < \frac{\sigma_b^2 - \sigma_{ab}}{\sigma_a^2 + \sigma_b^2 - 2 \sigma_{ab}}.$$

The inequality above implies that dE and $d\omega$ have opposite signs. Therefore if a VaR-constrained bank is hit by a negative shock on its equity, it will react by buying a bonds and selling the riskier b bonds (and vice versa if $dE > 0$). □

B Time-adapted CAViaR

We first review the CAViaR estimation method introduced by Engle and Manganelli (2004). Given a time series $\{Y_t\}_{t=1}^T$ the conditional Value-at-Risk is the time series $\{V_t^{(\alpha)}\}_{t=1}^T$ implicitly defined by the equation

$$\alpha = \text{Prob} \left[Y_t < -V_t^{(\alpha)} \mid \mathcal{I}_{t-1} \right]. \quad (22)$$

The quantile regression developed by Engle and Manganelli (2004) allows for the estimation of any parametric model for $V_t^{(\alpha)}$. In testing tail risk spillovers among financial time series, Hong et al. (2009) adopt the asymmetric slope model that specifies $V_t^{(\alpha)}$ as

$$V_t^{(\alpha)} = \beta_1 + \beta_2 V_{t-1}^{(\alpha)} + \beta_3 Y_{t-1}^+ + \beta_4 Y_{t-1}^-, \quad (23)$$

where the dependence of the β 's from α has been omitted to ease the notation and where the Y_{t-1}^+ (resp. Y_{t-1}^-) denotes the positive (resp. negative) part. Volatility clustering implies that a strongly positive significant β_2 is expected. Concerning β_3 and β_4 they define, respectively, the impact of positive and negative returns on future Value-at-Risk. Models such that reported in equation (23) are estimated in Engle and Manganelli (2004) through a quantile regression. In estimating model (23) on equity and bond returns we adopt the same procedure as well (see Section 6 of Engle and Manganelli, 2004, for more details). For our purposes, however, we have to take particular care since we are aimed at producing fully-causal time series of conditional Value-at-Risk of equity and bonds. Hence we have to fix two main issues. First, we want to have time series that are comparable with each other and then we have to mitigate the asynchronicity due to the different time zones of the countries in our dataset. Second, having in mind a Granger-type analysis, at each point in time only past information should be used to produce the Value-at-Risk estimates.

We start our procedure with a ‘‘training’’ window of six years. More precisely, to compute the initial value of the VaR time series, we use a window starting in $t_{\text{start}} = \text{January, 1, 2001}$ and ending at $t_{\text{end}}^{(0)} = \text{January, 1, 2006}$ ¹⁹. Then, for a given series either of equity log-returns or of bond yield variations, we estimate the model of Eq. (23). Hence, the final date $t_{\text{end}}^{(0)}$ is shifted by one month producing a new final date $t_{\text{end}}^{(1)} = t_{\text{end}}^{(0)} + 1 \text{ month}$ and the estimation procedure, now using data from t_{start} up to $t_{\text{end}}^{(1)}$ is repeated. The generic n -th time window is thus formed using data from t_{start} up to $t_{\text{end}}^{(n)} \equiv t_{\text{end}}^{(0)} + n \text{ months}$. The procedure is iterated over n until the end of the time stamp is reached, that is when the shifted final date $t_{\text{end}}^{(n)}$ occurs after February, 14, 2014, which is the last available day for all time series in the dataset.

The outcome of the procedure is the set of hit series $Z_{i,t}^B = \mathbf{1}_{\{Y_{i,t} < -V_{i,t}^{(\alpha)}(\hat{\boldsymbol{\theta}}_i)\}}$ where t spans across the 2-days time grid, $Y_{i,t}$ is either the series of bond yield variations or the series of equity log-returns, $V_{i,t}^{(\alpha)}(\hat{\boldsymbol{\theta}}_i)$ is the series of the estimated Value-at-Risk of $Y_{i,t}$, and $\hat{\boldsymbol{\theta}}_i = \{\beta_i, i = 1, \dots, 4\}$ is the parameter vector. We stress that, as mentioned in Section 5, while all the information available up to $t_{\text{end}}^{(n)}$ is used to estimate the parametric model (23), the networks of Granger causalities adopts only data in the three years prior to $t_{\text{end}}^{(n)}$, as a trade-off between the power of the causality tests and the necessity of isolating periods of financial distress.

In Tables 2-3 we report the estimated β 's and the corresponding p-value (in brackets) for, respectively, the 36 sovereign debt bonds and the 33 systemically important financial institutions in our dataset, estimated using the entire time span. Numbers in bold identify 99% significant parameters. The adequacy of the fit is measured by the p-value of the Dynamical Quantile (DQ) test of Engle and Manganelli (2004) reported in

¹⁹This choice does not affect country whose series is not available before January, 1, 2006. In fact, before proceeding to the estimation of the CAViaR model, we require that at least 100 (daily) observations of the series are present. This limitation is required since we initialize the CAViaR estimation with the first 10% of the series, hence a reasonable number of observations must be present in order to have a reliable estimate of $V_0^{(\alpha)}$ in (23).

Table 2: Parameter estimates of the model (23) for daily time series of sovereign bond yield variations.

	Country Bond											
	AUSTL	BUL	CAN	SWI	CZH	DNK	AUSTR	BEL	CRO	CYP	FIN	FRA
β_1	0.0031 (0.08)	0.0243 (0.00)	0.0002 (0.38)	-0.0007 (0.22)	0.0028 (0.10)	0.0014 (0.10)	0.0026 (0.21)	0.0028 (0.02)	0.0106 (0.06)	0.0002 (0.39)	0.0014 (0.08)	0.0019 (0.09)
β_2	0.9297 (0.00)	0.7972 (0.00)	0.9537 (0.00)	0.9357 (0.00)	0.8969 (0.00)	0.9391 (0.00)	0.9003 (0.00)	0.9016 (0.00)	0.7174 (0.00)	0.9215 (0.00)	0.9405 (0.00)	0.9046 (0.00)
β_3	0.0424 (0.21)	0.7558 (0.00)	0.0734 (0.01)	0.1573 (0.01)	0.1477 (0.01)	0.0901 (0.02)	0.1785 (0.00)	0.2220 (0.00)	0.3821 (0.00)	0.2101 (0.00)	0.0969 (0.01)	0.1876 (0.00)
β_4	0.1440 (0.02)	-0.2136 (0.00)	0.1125 (0.00)	0.1711 (0.01)	0.1952 (0.00)	0.1020 (0.00)	0.1256 (0.03)	0.0927 (0.03)	0.4085 (0.00)	0.1415 (0.24)	0.0952 (0.05)	0.1379 (0.06)
DQ	0.88	0.58	0.51	0.20	0.99	0.69	0.75	0.54	0.70	0.90	0.58	0.79
	GER	GRE	HUN	IRE	ITA	NET	POL	POR	SLO	SPA	SLK	UK
β_1	0.0017 (0.08)	0.0004 (0.43)	0.0052 (0.00)	0.0017 (0.25)	0.0016 (0.18)	0.0017 (0.02)	0.0013 (0.08)	-0.0024 (0.25)	0.0001 (0.35)	0.0014 (0.03)	0.0016 (0.19)	0.0013 (0.16)
β_2	0.9349 (0.00)	0.9278 (0.00)	0.8508 (0.00)	0.8015 (0.00)	0.8014 (0.00)	0.9354 (0.00)	0.9256 (0.00)	0.8605 (0.00)	0.9748 (0.00)	0.8865 (0.00)	0.9021 (0.00)	0.9599 (0.00)
β_3	0.0901 (0.01)	0.1826 (0.00)	0.2208 (0.00)	0.4397 (0.00)	0.5718 (0.00)	0.1031 (0.00)	0.1151 (0.00)	0.4621 (0.00)	0.0790 (0.00)	0.3688 (0.00)	0.2008 (0.00)	0.0389 (0.07)
β_4	0.1176 (0.01)	0.1140 (0.06)	0.2128 (0.00)	0.4085 (0.00)	0.2498 (0.17)	0.0932 (0.01)	0.1318 (0.00)	0.2358 (0.01)	0.0258 (0.08)	0.0885 (0.05)	0.1690 (0.01)	0.0775 (0.01)
DQ	0.08	0.47	0.08	0.75	0.55	0.92	0.18	0.61	0.71	0.90	0.88	0.79
	HK	JAP	MAL	NOR	NZ	ROM	SWED	SGP	TUR	RUS	US	VEN
β_1	0.0004 (0.28)	0.0005 (0.05)	0.0554 (0.00)	0.0002 (0.45)	0.0106 (0.02)	0.0033 (0.20)	0.0017 (0.03)	0.0042 (0.00)	0.0082 (0.12)	0.0039 (0.00)	0.0018 (0.14)	0.2404 (0.04)
β_2	0.9357 (0.00)	0.9299 (0.00)	-0.0764 (0.00)	0.9317 (0.00)	0.7891 (0.00)	0.8200 (0.00)	0.9425 (0.00)	0.8710 (0.00)	0.8486 (0.00)	0.8582 (0.00)	0.9129 (0.00)	0.4235 (0.07)
β_3	0.1028 (0.01)	0.2147 (0.00)	0.9976 (0.00)	0.0791 (0.01)	0.1304 (0.02)	0.4692 (0.05)	0.0308 (0.18)	0.1947 (0.00)	0.2900 (0.05)	0.2873 (0.00)	0.1596 (0.02)	0.1854 (0.09)
β_4	0.1472 (0.00)	0.0396 (0.06)	0.0320 (0.12)	0.2060 (0.00)	0.3472 (0.00)	0.3354 (0.00)	0.1366 (0.00)	0.1545 (0.00)	0.3157 (0.09)	0.2392 (0.00)	0.1321 (0.00)	0.4561 (0.06)
DQ	0.10	0.96	0.51	0.93	0.03	0.13	0.32	0.99	0.82	0.67	0.51	0.65

Note: Sovereign bonds are indicated by the corresponding country acronym. For each β we report in brackets the corresponding p-value. Parameters with a p-value smaller than 1% are reported in bold. The DQ-test is the p-value of the Dynamical Quantile test of Engle and Manganelli (2004). A value of the test below 1% means that the hypothesis that the model (23) fits the data is rejected with a confidence of 1%.

Tables 2-3 for each time series²⁰.

²⁰A p-value below 0.01 means that the hypothesis that model (23) is the true DGP is rejected with 99% confidence.

Table 3: Parameter estimates of the model (23) for the daily time series of equity log-returns of the 33 systemically important financial institutions.

	Equity Ticker											
	DEXB	CS	UBS	CBK	DB	SAN	BBVA	ACA	GLE	BNP	RBS	
β_1	0.0043 (0.03)	0.0010 (0.00)	0.0006 (0.02)	0.0015 (0.01)	0.0014 (0.02)	0.0006 (0.02)	0.0014 (0.00)	0.0005 (0.02)	0.0011 (0.00)	0.0009 (0.00)	0.0008 (0.01)	
β_2	0.7899 (0.00)	0.9032 (0.00)	0.9264 (0.00)	0.8597 (0.00)	0.8689 (0.00)	0.9290 (0.00)	0.8666 (0.00)	0.9429 (0.00)	0.9102 (0.00)	0.9083 (0.00)	0.9302 (0.00)	
β_3	0.1022 (0.00)	0.0806 (0.05)	0.0412 (0.29)	0.1245 (0.04)	0.1011 (0.00)	0.0054 (0.42)	0.1179 (0.00)	0.0419 (0.17)	0.0460 (0.15)	0.0435 (0.17)	0.0344 (0.21)	
β_4	0.4882 (0.00)	0.2532 (0.00)	0.2132 (0.00)	0.3574 (0.00)	0.3807 (0.00)	0.2421 (0.00)	0.3275 (0.00)	0.1576 (0.00)	0.2658 (0.00)	0.2760 (0.00)	0.2037 (0.00)	
DQ	0.64	0.96	0.79	0.80	0.50	0.63	0.83	0.60	0.84	0.58	0.67	
	STAN	HSBC	LLOY	BCS	UCG	NDA	RF	COF	GS	JPM	AXP	
β_1	0.0005 (0.05)	0.0007 (0.04)	0.0004 (0.00)	0.0005 (0.20)	0.0009 (0.01)	0.0013 (0.06)	0.0002 (0.25)	0.0009 (0.00)	0.0008 (0.01)	0.0007 (0.12)	0.0005 (0.06)	
β_2	0.8889 (0.00)	0.8446 (0.00)	0.9483 (0.00)	0.9335 (0.00)	0.9213 (0.00)	0.8503 (0.00)	0.9182 (0.00)	0.9011 (0.00)	0.8868 (0.00)	0.9197 (0.00)	0.8934 (0.00)	
β_3	0.1079 (0.15)	0.1907 (0.00)	-0.0315 (0.04)	0.0202 (0.34)	0.0456 (0.21)	0.1703 (0.07)	0.0938 (0.10)	0.0512 (0.13)	0.1250 (0.00)	0.0933 (0.12)	0.1225 (0.00)	
β_4	0.2857 (0.01)	0.4594 (0.00)	0.2205 (0.00)	0.2228 (0.00)	0.2109 (0.02)	0.3449 (0.00)	0.2214 (0.02)	0.2732 (0.00)	0.2981 (0.05)	0.1627 (0.02)	0.2832 (0.00)	
DQ	0.91	0.94	0.57	0.26	0.37	0.95	0.75	0.93	0.75	0.56	0.64	
	BBT	BAC	BK	C	FITB	MS	PNC	STT	STI	USB	WFC	
β_1	0.0003 (0.17)	0.0015 (0.07)	0.0008 (0.03)	0.0009 (0.00)	0.0005 (0.11)	0.0008 (0.00)	0.0001 (0.42)	0.0012 (0.00)	0.0004 (0.11)	0.0001 (0.42)	-0.0000 (0.50)	
β_2	0.9034 (0.00)	0.8398 (0.00)	0.9158 (0.00)	0.9072 (0.00)	0.9498 (0.00)	0.8991 (0.00)	0.9068 (0.00)	0.8554 (0.00)	0.9100 (0.00)	0.8396 (0.00)	0.9086 (0.00)	
β_3	0.1074 (0.01)	0.1337 (0.03)	0.0558 (0.05)	0.0483 (0.19)	0.0055 (0.44)	0.0588 (0.05)	0.0718 (0.24)	0.0979 (0.17)	0.0557 (0.28)	0.1839 (0.00)	0.1065 (0.00)	
β_4	0.3119 (0.00)	0.4188 (0.08)	0.2209 (0.00)	0.2843 (0.00)	0.1541 (0.00)	0.3133 (0.00)	0.3566 (0.00)	0.3985 (0.00)	0.2872 (0.00)	0.5056 (0.00)	0.2978 (0.00)	
DQ	0.23	0.70	0.61	0.15	0.96	0.86	0.71	0.66	0.87	0.93	0.22	

Note: Equities are indicated by the corresponding ticker. For each β we report in brackets the corresponding p-value. Parameters with a p-value smaller than 1% are reported in bold. The DQ-test is the p-value of the Dynamical Quantile test of Engle and Manganelli (2004). A value of the test below 1% means that the hypothesis that the model (23) fits the data is rejected with a confidence of 1%.

C Summary Statistics

This Appendix reports the summary statistics of the three blocks of data as introduced in Section 4.

Table 4 shows a summary statistics of the series of bond yield variations used for the construction of the causal network. Moreover, for each country we add the information coming from historical S&P long-term foreign currency ratings that is explicitly used in the flight-to-quality analysis of Section 6.

Similarly, Table 5 reports a summary statistics for the time series of equity log-returns, which, typically, are available for longer time periods. As for the bond series, all equity series end at February, 14, 2014. Note that in both Tables 4 and 5 the number of observations refers to the number of bond yield variations or equity log-returns in a two-days sub-sampled grid, and thus it is typically half of what it is expected from a daily time series. As anticipated in the main text, the sub-sampling is required in order to mitigate the effect of non-synchronous data (equities and bonds may refer to banks or countries that pertain to different parts of the world).

A third block of data is formed by CDS spreads. This part of the dataset is extensively adopted in Section 7.1 to define quality measures that are suitable for the proposed out-of-sample forecast exercise and that are (at least by construction) independent from the S&P country ratings. Table 6 reports a summary statistics for the CDS spread dataset. It is worth to mention that, despite all bond and equity series end at February, 14, 2014 (even if the starting date may vary from series to series), all CDS spread series end mostly at January, 13, 2014, with few exceptions. Moreover, while the summaries for bonds and equities in Tables 4 and 5 are related to one-lag differences (or log-differences) on a two-days subsampled grid, the summary for CDS spread is relative to the original daily grid. This is because, for a given country, the corresponding CDS spreads are used in Section 7.1 for the construction of a measure of bond quality in connection with the bond yield of the same country, hence there are no issues related to non-synchronicity of data. In this case we also rely on the original daily grid for bond yield as well.

Table 4: Summary statistic for five-years maturity sovereign bond yield variations.

Country	#Obs	First Yield	First Rating	Mean	Std	Kurt	Skew	Rat. Initial	Rat. Final
Australia	1707	18-Dec-2000	30-Apr-2003	-0.0011614	0.086692	4.1985	0.10344	AAA	AAA
Bulgaria	918	17-Jan-2007	28-Feb-2001	-0.0021423	0.26788	11.4813	-0.025379	BB	BBB
Canada	1563	06-Feb-2002	01-Jan-2003	-0.0018417	0.070579	5.061	0.15631	AAA	AAA
Switzerland	912	05-Feb-2007	05-Jun-2007	-0.0025198	0.048724	11.7672	-0.97995	AAA	AAA
Czech Rep.	1707	22-Dec-2000	26-Feb-2001	-0.0032534	0.072267	13.8517	-0.12909	AA	AA
Denmark	1707	18-Dec-2000	26-Nov-2002	-0.0024144	0.066528	4.6618	0.19488	AAA	AAA
Austria	1707	18-Dec-2000	29-May-2001	-0.0022451	0.069703	8.0128	0.14315	AAA	AA
Belgium	1707	18-Dec-2000	31-Jan-2001	-0.0021455	0.079765	17.4321	-0.28795	AA	AA
Croatia	1707	23-Feb-1999	04-Jan-2001	-0.0013913	0.073417	12.5473	0.81739	A	BBB
Cyprus	1248	08-Jul-2004	01-Feb-2002	0.00288	0.2969	87.3243	2.2977	A	BB
Finland	1044	31-Jan-2006	23-Apr-2001	-0.0022283	0.068877	4.6478	-0.0046261	AAA	AAA
France	1044	31-Jan-2006	12-Apr-2002	-0.002152	0.072982	6.0037	-0.17405	AAA	AA
Germany	1707	21-Dec-2000	02-Jul-2002	-0.0022529	0.068445	4.661	0.09215	AAA	AAA
Greece	1249	05-Jul-2004	02-Jan-2001	-0.0019649	0.87615	20.659	0.86456	AA	CC
Hungary	1707	26-Jan-1999	28-Feb-2001	-0.00073451	0.12418	127.6168	7.0949	A	BB
Ireland	1707	12-Jan-1999	01-Jan-2003	-0.0016136	0.14665	37.0698	-1.4267	AA	BBB
Italy	1050	12-Jan-2006	31-Jan-2001	-0.00060661	0.13384	17.3076	-0.59019	AA	BBB
Netherlands	1707	19-Dec-2000	31-Jul-2003	-0.0021168	0.066599	4.6625	0.13765	AAA	AAA
Poland	1437	27-Jan-2003	04-Jan-2001	-0.0016577	0.071337	26.6904	2.2572	BBB	A
Portugal	899	13-Mar-2007	07-Feb-2002	2.3583e-05	0.31568	25.2	0.29047	AA	BB
Slovenia	1707	22-Dec-2000	26-Apr-2001	-0.00095375	0.095164	46.5957	3.0981	A	A
Spain	1707	21-Dec-2000	31-Jan-2001	-0.0014353	0.11052	17.1361	-1.0718	AA	BBB
Slovakia	1260	04-Jun-2004	24-May-2001	-0.0026884	0.071821	12.7855	0.29499	BBB	A
United Kingdom	1707	21-Dec-2000	27-Oct-2004	-0.001923	0.068336	5.2821	-0.00094986	AAA	AAA
Hong Kong	1036	22-Feb-2006	02-Jul-2002	-0.0026838	0.070906	6.4891	0.33106	A	AAA
Japan	1707	21-Dec-2000	04-Jan-2001	-0.00044238	0.034059	8.13	0.5942	AA	AA
Malaysia	885	19-Apr-2007	23-Apr-2001	0.00050054	0.099712	245.6309	-0.70101	BBB	A
Norway	1707	27-Dec-2000	24-Oct-2003	-0.0021842	0.074505	7.0371	0.06374	AAA	AAA
New Zealand	892	02-Apr-2007	31-Jul-2003	-0.0025713	0.070055	5.8272	0.10122	AA	AA
Romania	912	02-Feb-2007	11-Feb-2002	-0.0024293	0.22238	39.5595	0.066911	B	BB
Sweden	1201	17-Nov-2004	04-Jan-2001	-0.001455	0.065942	6.4749	-0.35297	AA	AAA
Singapore	943	08-Nov-2006	18-Jul-2003	-0.0016856	0.061072	10.2324	0.57176	AAA	AAA
Turkey	1209	25-Oct-2004	19-Jan-2001	-0.007365	0.46916	84.4745	2.0867	B	BB
Russia	1707	03-Jan-2000	23-Apr-2001	-0.0076673	0.15226	23.197	0.51217	BB	BBB
United States	1153	31-Mar-2005	01-Dec-2003	-0.0023223	0.089286	5.4863	0.21661	AAA	AA
Venezuela	668	17-Dec-2008	26-Feb-2001	-0.0071942	0.33779	5.7275	0.13542	B	B

Note: Summary statistic for the bond sample. The first column is the name of the country. The second column indicates the total number of available observations, that is the total number of innovations (one-lag differences of bond yields) in the two-days sub-sampled grid. Despite the starting date may vary from series to series, the ending date is February, 14, 2014 for all the countries. The third column is the starting date of the series, that is when the first yield is available. The fourth column indicates when the first S&P rating is available. Mean, Std, Kurt and Skew indicate, respectively, mean, standard deviation, kurtosis and skewness of the corresponding series for the entire sample. Finally, the last two columns indicate, respectively, the initial and final S&P rating.

Table 5: Summary statistic for equity log-returns.

Ticker	#Obs	First Price	Mean	Std	Kurt	Skew
DEXB	1707	07-Oct-1999	-0.0035184	0.072745	15.2732	0.079262
CS	1643	27-Jun-2001	-0.00048936	0.038509	11.2898	0.32927
UBS	1681	14-Mar-2001	-0.00043545	0.03446	11.2177	-0.079069
CBK	1707	22-Apr-1998	-0.00054483	0.03749	10.5217	-0.052093
DB	1707	22-Apr-1998	-0.0015446	0.044453	10.2787	-0.41609
SAN	1707	22-Apr-1998	-0.00033222	0.031501	6.3791	0.062805
BBVA	1707	29-Dec-1999	-0.00034258	0.032301	6.446	-0.023647
ACA	1602	22-Oct-2001	-0.00035503	0.038336	8.253	-0.09679
GLE	1707	22-Oct-1998	-0.0002281	0.041138	7.6145	-0.1066
BNP	1707	22-Oct-1998	0.00017214	0.036591	8.1942	0.050653
RBS	1707	24-Mar-1999	0.00016734	0.032294	11.1185	-0.17534
STAN	1707	04-Feb-1999	-0.00021892	0.025165	16.1126	-0.86996
HSBC	1707	02-Feb-1999	-0.0010079	0.046532	24.338	-1.4578
LLOY	1707	02-Feb-1999	-0.00033476	0.046943	26.2879	0.26912
BCS	1707	10-Dec-1998	-0.0015173	0.053786	163.7187	-7.4101
UCG	1707	19-Aug-1998	-0.00099158	0.036409	9.1553	-0.11276
NDA	1707	22-Jun-1998	0.00038478	0.02919	8.9226	0.15603
RF	1707	14-Dec-1998	0.00034582	0.032113	13.4261	0.07461
COF	1707	26-Apr-1999	-1.6519e-06	0.029675	16.539	0.56877
GS	1707	14-Dec-1998	-0.00019598	0.042239	21.9987	-0.21996
JPM	1122	27-Jun-2005	4.1355e-05	0.030608	11.4117	-0.38165
AXP	1707	27-Apr-1999	6.3366e-05	0.043883	16.9162	-0.50853
BBT	1707	14-Dec-1998	-0.0013513	0.045729	21.7681	-0.57551
BAC	1707	18-Jun-1999	-0.00061847	0.052349	69.6301	2.3045
BK	1707	15-Jun-1999	0.00025387	0.032615	10.7813	-0.1228
C	1707	10-May-2000	0.00010819	0.033869	10.7917	-0.082293
FITB	1707	15-Dec-1998	-0.00042313	0.053264	70.7717	1.4406
MS	1707	16-Jun-1999	5.7451e-05	0.035257	36.7378	-1.5176
PNC	1187	27-Dec-2004	-0.0010434	0.052744	16.4883	0.27674
STT	1707	18-Jun-1999	7.2277e-05	0.039327	58.6034	-2.9936
STI	1707	23-Oct-1998	-0.00031073	0.041151	34.4266	1.1071
USB	1707	18-Jun-1999	0.00017	0.032021	26.1179	-0.4025
WFC	1707	14-Dec-1998	0.00029344	0.03257	20.9868	-0.35808

Note: Summary statistic for the equity sample. The first column indicates the bank ticker. The second column indicates the total number of available observations, that is the total number of innovations (one-lag log-differences of equity log-prices) in the two-days sub-sampled grid. Despite the starting date may vary from series to series, the ending date is February, 14, 2014 for all the equities. The third column is the starting date of the series, that is when the first price is available. Mean, Std, Kurt and Skew indicate, respectively, mean, standard deviation, kurtosis and skewness of the corresponding equity for the entire sample.

Table 6: Summary statistic for five-years maturity CDS spread variations.

Country	#Obs	First Obs.	Last Obs.	Mean	Std	Kurt	Skew
Australia	2697	30-Apr-2003	13-Jan-2014	8.4894e-07	0.00021067	26.3281	0.51716
Bulgaria	3342	28-Feb-2001	13-Jan-2014	-1.1356e-05	0.00088362	99.5856	-2.9087
Canada	2283	11-Sep-2003	13-Jan-2014	-1.3333e-06	0.00013204	34.555	-1.851
Switzerland	1308	05-Jun-2007	13-Jan-2014	-1.9349e-06	0.00025046	41.9152	-0.89803
Czech Rep.	3251	26-Mar-2001	13-Jan-2014	1.0436e-06	0.00034909	51.6964	0.45145
Denmark	2877	26-Nov-2002	13-Jan-2014	5.9399e-07	0.00018683	27.1692	0.44277
Austria	3295	29-May-2001	13-Jan-2014	9.8893e-07	0.00030032	37.8286	1.136
Belgium	3374	31-Jan-2001	13-Jan-2014	1.1869e-06	0.00038992	38.4293	-0.79271
Croatia	3398	04-Jan-2001	13-Jan-2014	1.792e-06	0.0007084	30.0985	-0.52374
Cyprus	2785	01-Jul-2002	13-Jan-2014	2.663e-05	0.0016567	99.2018	1.4096
Finland	2983	01-Jul-2002	13-Jan-2014	5.1689e-07	0.00011401	22.0623	0.92094
France	3061	12-Apr-2002	13-Jan-2014	1.5933e-06	0.00026709	23.924	-0.26315
Germany	2946	02-Jul-2002	13-Jan-2014	6.9075e-07	0.00014363	19.351	0.225
Greece	3036	02-Jan-2001	13-Jan-2014	0.00074927	0.02616	547.4371	13.5898
Hungary	2816	28-Feb-2001	29-Dec-2011	2.0483e-05	0.00083989	47.3315	1.7824
Ireland	2878	01-Jan-2003	13-Jan-2014	3.623e-06	0.0010063	45.6246	-0.62625
Italy	3379	31-Jan-2001	13-Jan-2014	4.3708e-06	0.0006985	26.7496	0.12638
Netherlands	2316	31-Jul-2003	13-Jan-2014	1.0806e-06	0.00019783	25.9883	1.0971
Poland	3372	04-Jan-2001	13-Jan-2014	1.0346e-06	0.00049855	35.175	0.12573
Portugal	3113	07-Feb-2002	13-Jan-2014	8.9085e-06	0.0014123	39.4631	-0.44806
Slovenia	3106	01-Feb-2002	13-Jan-2014	5.5031e-06	0.0004897	47.888	2.0997
Spain	3361	26-Feb-2001	13-Jan-2014	3.7635e-06	0.00070193	21.9508	-0.57469
Slovakia	3277	24-May-2001	13-Jan-2014	-1.9518e-06	0.00035572	31.4529	1.2219
United Kingdom	1991	20-Mar-2006	13-Jan-2014	1.2377e-06	0.00021594	15.3519	-0.2338
Hong Kong	2479	29-Aug-2003	13-Jan-2014	1.1084e-06	0.00029628	65.5134	3.1484
Japan	3380	04-Jan-2001	13-Jan-2014	9.6021e-07	0.00019432	59.3879	2.3455
Malaysia	3321	23-Apr-2001	13-Jan-2014	-1.3894e-06	0.00056761	159.714	1.0161
Norway	2657	24-Oct-2003	13-Jan-2014	4.1095e-07	0.00011122	47.1808	0.64787
New Zealand	2429	31-Jul-2003	13-Jan-2014	1.7824e-06	0.00028063	36.5511	0.46946
Romania	3051	21-Mar-2002	13-Jan-2014	-1.1689e-05	0.0010779	96.6887	-2.8478
Sweden	3008	29-May-2001	13-Jan-2014	3.2633e-07	0.00015686	31.691	0.50626
Singapore	1896	18-Jul-2003	23-Mar-2012	4.8426e-06	0.00016737	547.7068	17.8006
Turkey	3387	19-Jan-2001	13-Jan-2014	-8.7661e-06	0.0014769	72.3195	3.6072
Russia	3215	18-Sep-2001	13-Jan-2014	-1.8761e-05	0.0012738	88.5448	2.6698
United States	2581	01-Dec-2003	13-Jan-2014	1.0715e-06	0.00012687	20.2746	0.79051
Venezuela	3361	26-Feb-2001	13-Jan-2014	1.5273e-05	0.003007	57.8959	0.54851

Note:Summary statistic for the CDS spread sample. The first column is the name of the country. The second column indicates the total number of available observations, that is the total number of CDS spreads (one-lag differences) sampled at daily frequency. The third column is the starting date of the series, that is when the first CDS spread is available. The fourth column indicates when the series ends. Note that, differently from the bond yield and equity series, not all CDS series end at the same date, this is why the termination date is reported here for CDS spreads but omitted in Tables 4 and 5. Mean, Std, Kurt and Skew indicate, respectively, mean, standard deviation, kurtosis and skewness of the corresponding series for the entire sample.

D Granger Causality

Billio et al. (2012) apply the definition of causality as originally defined by Granger (1969). Consider two time series $\{Y_{1,t}\}_{t=1}^T$ and $\{Y_{2,t}\}_{t=1}^T$ and the two regressions

$$\begin{aligned} Y_{1,t+1} &= b_{1,1} Y_{1,t} + \varepsilon_{t+1} \\ Y_{1,t+1} &= b_{1,1} Y_{1,t} + b_{1,2} Y_{2,t} + \psi_{t+1}, \end{aligned} \quad (24)$$

where ε_{t+1} and ψ_{t+1} are i.i.d. normal shocks (with possibly non-zero mean). We say that $\{Y_{2,t}\}_{t=1}^T$ Granger-Causes the series $\{Y_{1,t}\}_{t=1}^T$, and we write $(2 \implies 1)$, at a confidence level α if the F-statistic of the two regressions

$$F = \left(\sum_{t=1}^T (\hat{\varepsilon}_t^2 - \hat{\psi}_t^2) \right) \left(\frac{\sum_{t=1}^T \hat{\psi}_t^2}{T-2} \right)^{-1}$$

is larger than the corresponding α -quantile of the F-distribution. Note that with this definition we are not testing for simultaneous causality. This choice is necessary in order to have a coherent comparison with the $Q_1(M)$ test, which does not check for simultaneous risk spillover. Note that, if the first of the equation (24) is the true data generating process, we have

$$\mathbb{E} \left[Y_{1,t} \mid \{Y_{1,t-k}\}_{k=1}^{t-1} \right] = b_{1,1} Y_{1,t}.$$

In particular we also have that

$$\mathbb{E} \left[Y_{1,t} \mid \{Y_{1,t-k}, Y_{2,t-k}\}_{k=1}^{t-1} \right] = b_{1,1} Y_{1,t} = \mathbb{E} \left[Y_{1,t} \mid \{Y_{1,t-k}\}_{k=1}^{t-1} \right].$$

On the other side, if the second of the (24) is the true data generating process we have that

$$\mathbb{E} \left[Y_{1,t} \mid \{Y_{1,t-k}\}_{k=1}^{t-1} \right] = b_{1,1} Y_{1,t} + b_{1,2} \mathbb{E} \left[Y_{2,t} \mid \{Y_{1,t-k}\}_{k=1}^{t-1} \right],$$

while

$$\mathbb{E} \left[Y_{1,t} \mid \{Y_{1,t-k}, Y_{2,t-k}\}_{k=1}^{t-1} \right] = b_{1,1} Y_{1,t} + b_{1,2} Y_{2,t} \neq \mathbb{E} \left[Y_{1,t} \mid \{Y_{1,t-k}\}_{k=1}^{t-1} \right].$$

In other words, when testing the Granger (1969)-causality we are testing the null

$$\mathbb{H}_G^0 : \mathbb{E} \left[Y_{1,t} \mid \{Y_{1,t-k}\}_{k=1}^{t-1} \right] = \mathbb{E} \left[Y_{1,t} \mid \{Y_{1,t-k}, Y_{2,t-k}\}_{k=1}^{t-1} \right] \quad (25)$$

against the alternative

$$\mathbb{H}_G^A : \mathbb{E} \left[Y_{1,t} \mid \{Y_{1,t-k}\}_{k=1}^{t-1} \right] \neq \mathbb{E} \left[Y_{1,t} \mid \{Y_{1,t-k}, Y_{2,t-k}\}_{k=1}^{t-1} \right]. \quad (26)$$

Hence, since $\text{Prob} \left[Y_{i,t} < -V_{i,t}^{(\alpha)} \mid \mathcal{F}_{t-1} \right] = \mathbb{E} [Z_{i,t} \mid \mathcal{F}_{t-1}]$, the main difference between the couple of null-alternative hypotheses in (25)-(26) and those in (3)-(4) is the substitution of the series $Y_{i,t}$ with the hit function $Z_{i,t}$, which signals the presence of tail events.

In order to avoid spurious detections (induced by heteroskedasticity) of causalities in the Granger (1969)-causality network we follow the procedure adopted by Billio et al. (2012) and we filter out a GARCH(1,1) model from data. That is, for each time series $Y_{i,t}$, we estimate the model

$$\begin{cases} Y_{i,t} &= \mu_i + \sigma_{i,t} \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim N(0, 1), \\ \sigma_{i,t}^2 &= \omega_i + \alpha_i (Y_{i,t-1} - \mu_i)^2 + \beta_i \sigma_{i,t-1}^2, \end{cases} \quad (27)$$

and then we normalize each time series by re-defining

$$Y_{i,t} \equiv \frac{Y_{i,t}}{\widehat{\sigma}_{i,t}},$$

where $\widehat{\sigma}_{i,t}$ is the estimated conditional volatility of model (27). This filtering is not required for the tail risk networks since heteroskedasticity is, in these cases, accounted for the parametric conditional Value-at-Risk model.

E Monte Carlo Estimation of the Confidence Intervals of the Centrality Measures

Figures 2 and 3 report the percentage of 5%-significant links in the bi-partite network of equity-bond, estimated in a rolling time-window of three years. In the ideal case in which the network is estimated via an infinite time series, both the causal test and the estimation of the VaR model in (23) or the GARCH model in (27) are immune by estimation errors. In this case we expect 5% of the equity-bond couples to be validated under the null of no-causal connection among them. In practice, finiteness of the sample and numerical errors in the estimation procedure could result in higher rejection rates under the null. In order to validate the statistical significance of the results shown in Figures 2 and 3 we perform a simple Monte Carlo experiment. For each equity and bond series in the dataset we randomly extract with replacement 400 observations from the corresponding time-series, hence forming a new set of 33 equity log-returns and 36 bond yield variations. The choice of 400 observations is dictated by the need of reproducing the sample size used for the estimation of the causal networks. After a new set of bootstrapped equity-bond data is formed, we proceed in the estimation of the three casual networks and we record the percentage of validated links at 5% confidence level. We iterate this procedure for 1000 times and we plot in Figure 10 the density plots of the three percentages of validated links, one for each type of causality. The horizontal lines in Figure 2 and 3 are in correspondence of the average percentage of 5%-validated links computed over the 1000 replications. These values are 6.8%, 6.6% and 5.2% for, respectively, the $(E_j \implies B_i^{\text{Buy}})$, $(E_j \implies B_i^{\text{Sell}})$ and $(E_j \implies B_i)$ causal networks. The slightly larger values found for the networks of tail causality is mainly a consequence of the unavoidable smaller statistics of the tail events.

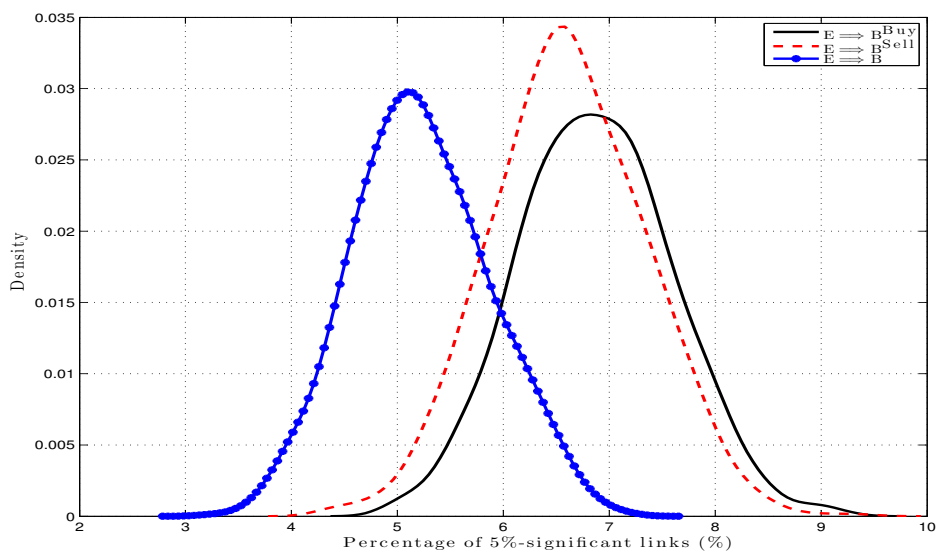


Figure 10: The black (continuous), the red (dotted) and the blue (with filled circles) lines represent, respectively, the density of the percentage of 5%-significant links according to the $(E_j \implies B_i^{\text{Buy}})$, $(E_j \implies B_i^{\text{Sell}})$ and $(E_j \implies B_i)$ causality test. The density is computed over 1000 replications of a bootstrap procedure in which, for each replication, a new set of 33 equity log-returns and 36 bond yield variations is formed by randomly extracting with replacement 400 observations from the original time series. Densities are normalized to have an integral equal to one.

F Results on other proxies

Nested models comparison with dependent variable $\bar{S}_{i,t}$.

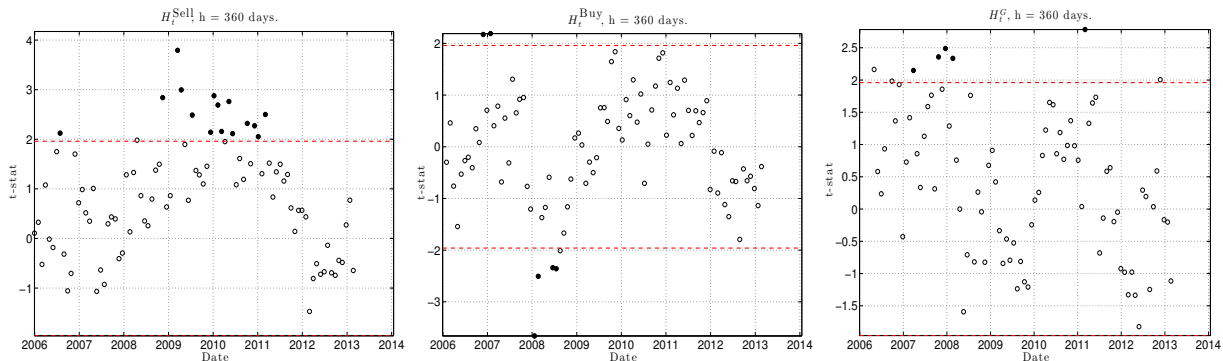


Figure 11: This figure depicts, for each time-window $t = 1, \dots, 99$ whose closure is at day e_t (reported in the horizontal axis), the t-statistic of β_t in (15) where the dependent variable $\mathcal{Y}_{i,t}$ is the ranking of the variable $\bar{S}_{i,t}$ defined in (13) and computed in the forecasting horizon $(e_t, e_t + H]$, with $H = 360$. Each column corresponds to a different regressor $\mathcal{X}_{i,t}$ in (15). More precisely, $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}^{B+}$ for the first, $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}^{B-}$ for the second and $\mathcal{X}_{i,t} = \mathcal{H}_{i,t}$ for the third. A black point is reported whenever the regression (16) is significantly better (according to the log-likelihood ratio test of Vuong (1989) with 95% confidence) than the simple auto-regressive model (14). Horizontal red dotted lines are in correspondence of 95% confidence level of the standard Normal distribution.

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